Math 502
Problem Set #4
Due October 21

Problem 1: Exercise 6.8 in the Marker notes.

Problem 2: Exercise 6.9 in the Marker notes.

Problem 3: Exercise 6.10 in the Marker notes.

Problem 4: Exercise 6.11 in the Marker notes.

Problem 5: Exercise 6.13 in the Marker notes.

Problem 6: Write register machine programs for the following functions:
(a) max(a, b)
(b) a − b := max(a − b, 0).

Problem 7: Show that the following functions are primitive recursive:
(a) p : \mathbb{N} → \mathbb{N} defined by
   \[ p(x) = \begin{cases} 
   1 & \text{if } x \text{ is even} \\
   0 & \text{otherwise} 
   \end{cases} \]
(b) f : \mathbb{N} → \mathbb{N} defined by
   \[ f(x) = \begin{cases} 
   1 & \text{if Goldbach’s conjecture is true} \\
   0 & \text{otherwise} 
   \end{cases} \]

Problem 8:
(a) Suppose that \( R \subseteq \mathbb{N}^k \) is finite. Prove that \( R \) is primitive recursive.
(b) Suppose that \( f : \mathbb{N} → \mathbb{N} \) is eventually constant, that is, there is \( k \in \mathbb{N} \) such that, for all \( m, n \geq k \), we have \( f(m) = f(n) \). Prove that \( f \) is primitive recursive.
(c) Suppose that \( f : \mathbb{N} → \mathbb{N} \) is primitive recursive and increasing, i.e. \( f(n) < f(n + 1) \) for all \( n \in \mathbb{N} \). Prove that \( f(\mathbb{N}) \) is primitive recursive.
(d) Suppose that \( f : \mathbb{N} → \mathbb{N} \) is recursive, nondecreasing, i.e. \( f(n) \leq f(n + 1) \) for all \( n \in \mathbb{N} \), and unbounded. Prove that \( f(\mathbb{N}) \) is recursive.
(e) Suppose that \( f : \mathbb{N}^k → \mathbb{N} \) has finite domain. Prove that \( f \) is recursive.