Problem 1: Exercise 8.16 in the Marker notes.

Problem 2: Suppose that $f : \mathbb{N} \to \mathbb{N}$ is a computable function. Prove that $\bigcup_{n \in \mathbb{N}} W_{f(n)}$ is recursively enumerable.

Problem 3: Show that there is a computable partial function $f : \mathbb{N} \to \mathbb{N}$ such that, whenever $W_x \neq \emptyset$, then $f(x) \in W_x$.

Problem 4: Show that $\{ x \in \mathbb{N} : \phi_x(x) = 0 \}$ is recursively enumerable, but not recursive.

Problem 5: Show that there is a computable function $g : \mathbb{N}^2 \to \mathbb{N}$ such that, for all $x, y \in \mathbb{N}$, we have $W_{g(x,y)} = W_x \cup W_y$.

Problem 6: Show directly that $K \leq_m \{ x : W_x \text{ is infinite} \}$.

Problem 7: Show that $\{(x, y) : W_x \subseteq W_y\}$ is $\Pi^0_2$.

Problem 8: Set $Z := \{ x : \phi_x(y) = 0 \text{ for all } y \in \mathbb{N} \}$.

1. Show that $Z$ is $\Pi^0_2$.

2. Show that $Z$ is not $\Sigma^0_2$. 