Required Part:

0. Read §4b Tests of Hypotheses and Interval Estimation; §4c Problems of a Single Sample; §4d One-way Classified Data; and §4e Two-way Classified Data.

1. Consider the multiple regression model:

\[ y_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_{m-1} x_{m-1,i} + \epsilon_i, \]

\[ i = 1, \ldots, n \] with \( \epsilon_i \)'s iid from \( N(0, \sigma^2) \).

(i) Give a sufficient condition on the \( x_{ji} \)'s under which \( \beta_0, \beta_1, \ldots, \beta_{m-1} \) are all estimable.

(ii) If \( \hat{\beta}_j \) denotes a least square estimate of \( \beta_j \), then show that

\[ \sum_{i=1}^{n} \left[ y_i - \left( \hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \cdots + \hat{\beta}_{m-1} x_{m-1,i} \right) \right] = 0. \]

2. Let \( t_1, \ldots, t_k \) be unbiased estimators of a single parameter \( \theta \) and \( \text{cov}(t_i, t_j) = \sigma_{ij} \). Suppose \( \Sigma = (\sigma_{ij}) \) is positive definite. Find the linear function of \( t_1, \ldots, t_k \) unbiased for \( \theta \) and having minimum variance.

*Hint:* See (ii) of §1f.1 on page 60.

3. Show that if \( t_1, \ldots, t_k \) are unbiased minimum variance estimators of the parameters \( \theta_1, \ldots, \theta_k \), respectively, then \( c_1 t_1 + \cdots + c_k t_k \) is the unbiased minimum variance estimator of \( c_1 \theta_1 + \cdots + c_k \theta_k \).

*Hint:* Show that if \( T \) is an unbiased minimum variance estimator of \( \theta \), \( D \) is another estimator such that \( E(D) = 0 \), then \( \text{Cov}(T, D) = 0 \).