Homework 5

1) Consider the set $S$ of all ordered $k$-tuples $A = (A_1, \ldots, A_k)$ of subsets of $[n]$. Determine
$$\sum_{A \in S} |A_1 \cup \ldots \cup A_k|.$$

2) Give an explicit bijection between the set of plane rooted trees with $n$ vertices and the set of plane rooted binary trees with $n$ leaves.

3) Let $F_n(x)$ denote the expansion of $(1 - x^n)^{-\mu(n)/n}$ in a power series. Also consider the expansion of $e^x$ as a power series. Prove that
$$e^x = \prod_{n=1}^{\infty} F_n(x)$$
is true as a relation between formal power series.

4) Find the exponential generating function for the number of symmetric $n$ by $n$ permutation matrices.

5) Find the exponential generating function for the number of labeled 2-regular graphs on $n$ vertices.