1. (10 points) Convert the following base 10 numbers to binary:
   (a) (5 points) \((11.75)_{10}\)
   (b) (5 points) \((23/8)_{10}\)

2. (10 points) Convert the following binary numbers to base 10:
   (a) (5 points) \((11010.1)_{2}\)
   (b) (5 points) \((101.001)_{2}\)

3. (15 points) Consider the Fixed–Point Iterations (FPIs)
   (i.) \(x \rightarrow 2 + \frac{1}{x+2}\),
   (ii.) \(x \rightarrow \frac{x+4}{x+1}\),
   (iii.) \(x \rightarrow \frac{x}{2} + \frac{5}{2x}\).
   (a) (8 points) Which of the FPIs converge to \(\sqrt{5}\)?
   (b) (7 points) Rank the ones that converge from fastest to slowest.

4. (15 points) Consider the interpolating polynomial for \(f(x) = e^x\) with \((N + 1)\)–many equally spaced interpolation nodes on \([-L, L]\)

   \[x_n = -L + \frac{2Ln}{N}, \quad n = 0, \ldots, N.\]

   Find an upper bound for the interpolation error at
   (a) (5 points) \(x = 1/2\) if \(L = 1\) and \(N = 2\),
   (b) (5 points) \(x = 1\) if \(L = 2\) and \(N = 4\),
   (c) (5 points) \(x = 0\) for general \(L\) and \(N\).
5. (20 points) [COMPUTATIONAL PROBLEM] The function

\[ f(x) = x^4 + x^3 - 3x^2 - 5x - 2 \]

has roots at \( r_1 = 2 \) and \( r_2 = -1 \).

(a) (5 points) Apply Newton’s Method to this problem with initial guess \( x_0 = 1.9 \). How many iterations are required to approximate the root \( r_1 = 2 \) with an accuracy of \( 10^{-5} \)?

(b) (5 points) Apply Newton’s Method to this problem with initial guess \( x_0 = -1.1 \). How many iterations are required to approximate the root \( r_2 = -1 \) with an accuracy of \( 10^{-5} \)?

(c) (10 points) Apply Modified Newton’s Method to this problem with initial guess \( x_0 = -1.1 \). How many iterations are required to approximate the root \( r_2 = -1 \) with an accuracy of \( 10^{-5} \)?

6. (30 points) [COMPUTATIONAL PROBLEM] Consider the sparse linear system \( A\vec{x} = \vec{b} \) where

\[
A = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
1 & -2 & 1 & 0 & 0 & \cdots & 0 \\
0 & 1 & -2 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 1 & -2 & 1 & 0 \\
0 & \cdots & 0 & 0 & 1 & -2 & 1 \\
0 & \cdots & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix} \in \mathbb{R}^{N \times N}, \quad \vec{b} = \begin{pmatrix}
0 \\
-2/h^2 \\
-2/h^2 \\
\vdots \\
-2/h^2 \\
-2/h^2 \\
0 \\
\end{pmatrix} \in \mathbb{R}^N,
\]

and \( h = 1/(N-1) \). The solution of this linear system is

\[
\vec{x} = \begin{pmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{N-1} \\
x_N \\
\end{pmatrix}^T \in \mathbb{R}^N, \quad x_n = nh(1 - nh).
\]

(a) (10 points) Solve this linear system with the Jacobi Method for \( N = 10 \). How many iterations are required to realize a relative forward error (in the infinity norm) of \( \tau = 10^{-4} \)?

(b) (10 points) Repeat part (a) with the Gauss–Seidel Method.

(c) (10 points) Repeat part (a) with the SOR Method using the seventeen (17) values

\[ \omega = 0.2, 0.3, 0.4, \ldots, 1.6, 1.7, 1.8. \]