1. Write the general power, exponential and log rules for both differentiation and integration.

**SOLUTION:**

\[
\frac{d}{dx} (f(x))^n = nf'(x)(f(x))^{n-1} \quad \text{general power rule for differentiation}
\]

\[
\int f'(x)(f(x))^n \, dx = \frac{(f(x))^{n+1}}{n+1} + C \quad \text{general power rule for integration}
\]

\[
\frac{d}{dx} e^{f(x)} = f'(x)e^{f(x)} \quad \text{general exponential rule for differentiation}
\]

\[
\int f'(x)e^{f(x)} \, dx = e^{f(x)} + C \quad \text{general exponential rule for integration}
\]

\[
\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)} \quad \text{general log rule for differentiation}
\]

\[
\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + C \quad \text{general log rule for integration}
\]
2. Use logarithmic differentiation to find \( \frac{df}{dx} \) where \( f(x) = (9x + 1)^x \).

**SOLUTION:**

Taking the natural logarithm of both sides of the equation

\[ f(x) = (9x + 1)^x \]

yields:

\[ \ln(f(x)) = \ln((9x + 1)^x) \]

now recall that \( \ln(a^b) = b \ln(a) \), so we can re-write the right-hand side as shown below:

\[ \ln(f(x)) = x \ln(9x + 1) \]

now we differentiate both sides of the above to get:

\[ \frac{d}{dx} \ln(f(x)) = \frac{d}{dx}(x \ln(9x + 1)) \]

now on the left-hand side we apply the general log rule for differentiation, and on the right hand side we apply the product rule for differentiation to get:

\[ \frac{f'(x)}{f(x)} = (\frac{d}{dx}x) \ln(9x + 1) + x(\frac{d}{dx}\ln(9x + 1)) \]

now on the right hand side we evaluate the derivatives to get:

\[ \frac{f'(x)}{f(x)} = \ln(9x + 1) + x \frac{9}{9x + 1} \]

and finally we multiply both sides by \( f(x) \) (which is \( (9x + 1)^x \)) to get our final result:

\[ f'(x) = (\ln(9x + 1) + \frac{9x}{9x + 1})(9x + 1)^x. \]
3. Evaluate the three definite integrals below.  
(Use the general power rule, exponential rule or log rule. Do not use the substitution method.)

(a)  \[ \int_{3}^{5} 2x(x^2 + 1)^6 \, dx \]

**SOLUTION:**

We apply the general power rule for integration from question (1), noting that we are choosing \( f(x) = x^2 + 1 \), and that \( f'(x) = 2x \) is already present in the integrand, so there is no need to apply a "multiply by one trick" to obtain the differential. With this in mind, it is clear that:

\[
\int_{3}^{5} 2x(x^2 + 1)^6 \, dx = \left[ \frac{(x^2 + 1)^7}{7} \right]_{3}^{5} = \frac{(5^2 + 1)^7}{7} - \frac{(3^2 + 1)^7}{7} = \frac{26^7 - 10^7}{7}
\]

(b)  \[ \int_{3}^{5} 3e^{3x+1} \, dx \]

**SOLUTION:**

We apply the general exponential rule for integration from question (1), noting that we are choosing \( f(x) = 3x + 1 \), and that \( f'(x) = 3 \) is already present in the integrand, so once again there is no need to apply a "multiply by one trick" to obtain the differential. With this in mind, it is clear that:

\[
\int_{3}^{5} 3e^{3x+1} \, dx = [e^{3x+1}]_{3}^{5} = e^{3(5)+1} - e^{3(3)+1} = e^{16} - e^{10}
\]

(c)  \[ \int_{3}^{5} \frac{1}{6x+1} \, dx \]

**SOLUTION:**

We apply the general log rule for integration from question (1), noting that we are choosing \( f(x) = 6x + 1 \). In this case however, we note that \( f'(x) = 6 \) is not present in the integrand, so we do need to apply a "multiply by one trick" to obtain the differential form \( 6dx \):

\[
\int_{3}^{5} \frac{1}{6x+1} \, dx = \int_{3}^{5} \frac{\frac{1}{6}}{6x+1} \, dx = \frac{1}{6} \int_{3}^{5} \frac{6}{6x+1} \, dx
\]

Now it is clear that the right-most integral in the line above is:

\[
= \frac{1}{6} \left[ \ln |6x + 1| \right]_{3}^{5} = \frac{1}{6} (\ln |6(5) + 1| - \ln |6(3) + 1|) = \frac{1}{6} (\ln(31) - \ln(19)) = \frac{1}{6} (\ln\left(\frac{31}{19}\right))
\]