MCS 549 – Foundations of Data Science
Fall 2022
Problem Set 2

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Due: 11/2/22 at the beginning of class

Instructions: Atop your problem set, please write your name and list your collaborators.

Problems

1. A Markov chain is said to be symmetric if for all i and j, $p_{ij} = p_{ji}$. What is the stationary distribution of a connected symmetric Markov chain? Prove your answer.

2. Given a Markov chain on an undirected graph, we modify the chain as follows: at the current state, we stay there with probability 1/2; with the other probability 1/2, we move as in the old chain. Show that the new chain has the same stationary distribution.

3. Given the set of integers $\{1, 2, \ldots, n\}$, what is the expected number of draws d with replacement until the integer 1 is drawn? What is the expected number of draws until every integer from the set is drawn? (This is needed for the expected cover time of K_n.)

4. What is the hitting time h_{uv} for two adjacent vertices on a cycle of length n? What is the hitting time if edge (u, v) is removed?

5. What is the escape probability of a random walk starting at the root of an infinite binary tree? Show how you arrived at your answer.

6. Prove that two independent random walks starting at the origin on a two dimensional lattice will eventually meet with probability one.