MCS 541 – Computational Complexity
Spring 2023
Problem Set 1

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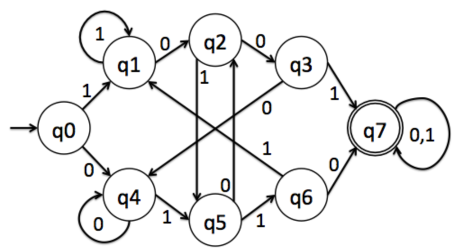
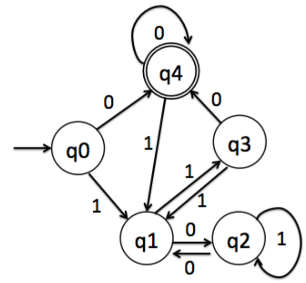
Due: 1/23/23 at the beginning of class

1. For the two following DFAs, explain in words the languages $L_{1.a}$ and $L_{1.b}$ that they recognize.

a. $M_{1.a}$



b. $M_{1.b}$



2. Let $L_{2.a}$ and $L_{2.b}$ be regular languages. Show that $\{x|x \in L_{2.a}, x \notin L_{2.b}\}$ is also regular.

3. Let $L_3 = \{0^n1^n \mid n \geq 1\}$. Draw the state diagram for a Turing Machine that decides L_3. All transitions not depicted will be assumed to go to the reject state, which may or may not be drawn. Note: knowing how to program TMs is not an important, but doing it at least once is a worthwhile execrise.

4. The goal of this problem is to find an uncomputable (or "incalculable") function. This function, $S(n)$, will grow so fast that even Turing machines cannot keep up with it.

a. Let $L_{4.a} = \{\langle M \rangle \mid M$ is a TM and M halts on the input ε (i.e. no input)$\}$. You can assume the input alphabet to be $\Sigma = \{0, 1\}$, which is sufficient to encode M. Is $L_{4.a}$ recognizable? Is $L_{4.a}$ decidable? Prove your answers correct.

b. Let $\mathcal{H}_n$ be the set of n-state TMs that eventually halt when run on the empty input. Let $S(n)$ be the maximum number steps a TM in $\mathcal{H}_n$ can take. Let

$$L_{4.b} = \{m \mid m \in \{0,1\}^* \text{ s.t. } m = S(n) \text{ for some } n \geq 1\}.[1]$$

Prove that $L_{4.b}$ is not decidable.

5. Consider the language L_5, defined as follows: $L_5 = \begin{cases} \{1\} & \text{if } L_{4.b} \text{ contains infinitely many primes} \\ \{0\} & \text{otherwise.} \end{cases}$
Is L_5 decidable? Why or why not?

[1] Interpret m as a binary number.