MCS 541 – Computational Complexity
Spring 2023
Problem Set 4[*]

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Due: 3/29/23 at the beginning of class

1. In class we proved that if $\mathbf{P} = \mathbf{NP}$ then $\mathbf{PH} = \mathbf{P}$. Extend this reasoning to prove that for every $i \geq 1$, if $\mathbf{\Sigma_i^p} = \mathbf{\Pi_i^p}$ then $\mathbf{PH} = \mathbf{\Sigma_i^p}$.

2. Define

$$\text{FACT} = \{\langle n, k \rangle | \ n \text{ has a prime factor that is smaller than } k\}$$

as before. Prove that if FACT is $\mathbf{NP}$-hard then $\mathbf{PH} = \mathbf{NP}$.

3. Prove that $\mathbf{IP} \subseteq \mathbf{PSPACE}$. *(Note that we discussed this in class but only formally argues about the other direction.)*

4. Let $\mathbf{IP'}$ be the class $\mathbf{IP}$ redefined so that the $2/3$ constant is changed to 1 and the $1/3$ constant is changed to 0. Prove that $\mathbf{IP'} = \mathbf{NP}$. *(Note that to bring* $\mathbf{IP}$ *down to* $\mathbf{NP}$*, it is actually sufficient to just change the* $1/3$ *constant to* 0 *without changing the* $2/3$ *constant.)*

5. Define a TM M to be *oblivious* if its head movements do not depend on the input, but only on the input length. That is, M is oblivious if for every $x \in \{0,1\}^*$ and $i \in \mathbb{N}$, the location of each of M's heads at the ith step of execution on input x is only a function of $|x|$ and i. Show that for any $L \in \mathbf{P}$, L is computable in polynomial time by an oblivious TM.

[*]Many of these problems are modifications of exercises that appear in Arora-Barak.