Undergraduate Mathematics Symposium

Saturday, October 1, 2011
University of Illinois at Chicago

Schedule of Activities

8:15am – 8:55am	Sign-in and coffee in SEO 300
Morning Session – Plenary lectures, 50 minutes each, SEO 636	
8:55am	Opening remarks
9:00am	Yvonne Lai (U Michigan) **Decomposing Polyhedra**
10:00am	Dhruv Mubayi (UIC) **From Ramsey theory to arithmetic progressions and hypergraphs**
11:00am	Christopher Leininger (UIUC) **Geometry and dynamics of surface homeomorphisms**
Noon – 1:30pm	Lunch in SEO 300
Afternoon Session 1 – Student lectures, 20 minutes each, SEO 636	
1:30pm	Sean McAfee (UIC) **On improving Cayley's theorem for groups of order p^4**
2:00pm	Jonathan Gleason (U Chicago) **The F^*-algebraic Formulation of Quantum Mechanics**
2:30pm	Christopher Perez (CalTech) **Siegel modular varieties: arithmetic invariants and cusps**
2:50pm – 3:30pm	Coffee break in SEO 300
Afternoon Session 2 – Student lectures, 20 minutes each, SEO 636	
3:30pm	In-Jee Jeong (Brown) **Bipartite Graphs, Quivers, and Cluster Variables**
4:00pm	Anita Thomas (IIT) **Sampling Within k-Means Algorithm to Cluster Large Datasets**
4:30pm	Jahan Claes (U Chicago) **Spectral Rigidity on Flat Tori**
5:00pm	Joshua Jay Herman (UIC) **Introduction to topological quantum computation**

Abstracts of Plenary Lectures

Yvonne Lai (U Michigan)

Decomposing Polyhedra

Suppose you are given two polygons of equal area. Can you slice up one polygon with scissors, rearrange the pieces, and obtain the other polygon? What if we ask the analogous problem of polyhedra? How we answer this question depends on how we define slicing and rearranging.

Starting from natural but different definitions leads to radically disparate answers. We will examine these questions from several perspectives, winding our way from Euclid's Elements to foundational mathematics of the 20th century.

Christopher Leininger (UIUC)

Geometry and dynamics of surface homeomorphisms

Surfaces are fundamental objects in mathematics arising in a number of different fields. Although compact surfaces were classified 150 years ago, they continue to be central to many areas of mathematics. In this talk, I'll give some indication of Thurston's classification theorem for surface homeomorphisms, primarily through examples. Then I'll focus on the "generic" type, the *pseudo-Anosov* homeomorphisms. I will explain some of the connections between geometry, topology and dynamics for this class of homeomorphisms.

Dhruv Mubayi (UIC)

From Ramsey theory to arithmetic progressions and hypergraphs

I will begin by giving a brief introduction to Ramsey theory, in layman's terms, and describing one of the most well-known open problems in this area. This will naturally lead to a similar question about arithmetic progressions in the integers. Finally, we will see a glimpse of how these problems about arithmetic progressions are solved by modern combinatorial methods using hypergraphs.

Abstracts of Student Lectures

Jahan Claes (U Chicago)
Spectral Rigidity on Flat Tori

We define an n-dimensional flat torus as both a topological and metric space, and discuss basic notions of spectra we may define on tori. We then sketch a proof of a preliminary rigidity result: that isospectral 2-tori are isometric. Given time, we will also define higher-dimensional spectra, and outline arguments for higher-dimensional rigidity. No background is necessary, although a knowledge of topology may prove useful. The material in this talk is adapted from my University of Chicago REU project.

Jonathan Gleason (U Chicago)
The F^*-algebraic Formulation of Quantum Mechanics

There is an approach to quantum mechanics known as the C^*-algebraic formulation of quantum mechanics that takes as an assumption that the observables in quantum mechanics are exactly the self-adjoint elements of a separable, unital C^*-algebra. By the celebrated Gelfand-Naimark Theorem, this abstract C^*-algebra is isometrically isomorphic to a closed subalgebra of the C^*-algebra of bounded operators on a separable Hilbert space. This, however, is problematic, as unbounded operators are absolutely essential in quantum mechanics.

To get around this problem, I introduce the notion of what I call an F^*-algebra, which is a generalization of a C^*-algebra in the same vein that a Fréchet space is the generalization of a Banach space. I explain how it is more natural to take observables as the self-adjoint elements of a separable, unital F^*-algebra, and I then use the Gelfand-Naimark Theorem to show that all separable, unital F^*-algebras are isomorphic an F^*-algebra of closed, densely defined linear operators on a separable Hilbert space, thereby showing that this formulation of quantum mechanics is equivalent to the usual formulation given in most textbooks. I did this work during the 2011 REU at the University of Chicago.

Joshua Jay Herman (UIC)
Introduction to topological quantum computation

What will be covered is the basics of quantum computation such as the different models of computation. The topics in order of coverage will be classical computation with matrices, probabilistic models of computation, the quantum computation model without topology or basically quantum computing at the gate level, then what will be covered is the Quantum Turing Machine and quantum gate models of computation, lastly we will cover current research topics in quantum computation such as category theory and topological quantum computation and open problems such as parallel quantum computation.

(Abstracts continue on next page.)

In-Jee Jeong (Brown)

Bipartite Graphs, Quivers, and Cluster Variables

In this paper we explore connections between formulas for certain combinatorial and algebraic objects. In particular, given a planar bipartite graph G, we consider the cluster algebra A (which is usually of infinite mutation type) corresponding to a quiver obtained from the dual graph of G. We then obtain formulas for certain cluster variables in A in terms of perfect matchings of subgraphs of G. Such subgraphs look like trees; locally they look like snakes with bridges connecting them. Finally, in the case of a spine snake with some trees attached to its edges we can obtain combinatorial formulas through superpositions. This work was completed at a 2011 REU program at the University of Minnesota.

Sean McAfee (UIC)

On improving Cayley's theorem for groups of order p^4

A group G of order n is always isomorphic to a subgroup of the symmetric group S(n). In what cases is there an integer $m<n$ such that G is isomorphic to a subgroup of S(m)? I will describe my calculation of such an m for all 15 (up to isomorphism) groups of order p^4, $(p>3)$.

Christopher Perez (CalTech)

Siegel modular varieties: arithmetic invariants and cusps

Siegel modular forms are a generalization of elliptic modular forms to several complex variables. As in the case of elliptic modular forms, there is an associated modular variety known as a Siegel modular variety. We attempt to generalize the notion of modular symbols to Siegel modular varieties and use these to generate a basis for Siegel modular forms.

Anita Thomas (IIT)

Sampling Within k-Means Algorithm to Cluster Large Datasets

Due to current data collection technology, our ability to gather data has surpassed our ability to analyze it. In particular, k-means, one of the simplest and fastest clustering algorithms, is ill-equipped to handle extremely large datasets on even the most powerful machines. Our new algorithm implements sampling within k-means to reduce the amount of data analyzed, thus decreasing run-time. We perform a simulation study to compare our sampling based k-means to the standard k-means algorithm by analyzing both speed and accuracy. Results show that our algorithm is significantly more efficient than the existing algorithm with comparable accuracy. This research was completed as part of the REU Site *Interdisciplinary Program in High Performance Computing* at the University of Maryland, Baltimore County.