

Math 215 - Introduction to Advanced Mathematics

Sets and Functions Worksheet

Fall 2017

1. Prove the following: Let $f : X \rightarrow Y$ be a function. Then f is invertible if and only if it is a bijection. Moreover, its inverse function is unique.

2. Prove that

$$(A \cup B) \cap (C \cup D) = (A \cap C) \cup (A \cap D) \cup (B \cap C) \cup (B \cap D)$$

only using the facts that unions/intersections are associative and commutative and that union distributes over intersection and intersection distributes over union.

3. Prove that if $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are surjections, then the function

$$g \circ f : X \rightarrow Z$$

is also a surjection.

4. Let U be a set. For any subset $A \in \mathcal{P}(U)$, define the function $\chi_A : U \rightarrow \{0, 1\}$ by

$$\chi_A(x) = \begin{cases} 0 & x \notin A \\ 1 & x \in A \end{cases}$$

For what set $C \in \mathcal{P}(U)$ is

$$\chi_C(x) = \chi_A(x)\chi_B(x)$$

for all $x \in U$? For what set $C \in \mathcal{P}(U)$ is

$$\chi_C(x) = \chi_A(x) + \chi_B(x) - \chi_A(x)\chi_B(x)$$

for all $x \in U$?

5. For each $r \in \mathbb{Q}$, let

$$D_r = \left(\frac{1}{2} - r, \frac{1}{2} + r \right).$$

Find $\bigcup_{r \in \mathbb{Q}} D_r$ and $\bigcap_{r \in \mathbb{Q}} D_r$. What happens if we change \mathbb{Q} to \mathbb{Q}^+ ?

6. For each $r \in \mathbb{Q}$, let K_r be the set containing all real numbers except for r . Find $\bigcup_{r \in \mathbb{Q}} K_r$ and $\bigcap_{r \in \mathbb{Q}} K_r$.