Math 215 - Introduction to Advanced Mathematics

Problem Set 10

Spring 2018
Due in class on Friday, April 20

For each of the following questions give your answer and then explain the reasons why your answer is correct using full sentences.

1. Show that if $n+1$ integers are chosen from the set $\{1, 2, \ldots, 2n\}$, then there are always two which differ by 1.

2. There are 100 people at a party. Any two people are either friends or they're not friends - nothing ambiguous and no one-sided friendships. **(a)** Assume that each person has a nonzero, even number of friends at the party (not including themselves). Prove that there are three people with exactly the same number of friends. **(b)** Is this still true if each person has an even (possibly zero) number of friends? Why or why not?

3. There are 10 people in a room with ages ranging from 1 to 60. Prove that we can always find two groups of people, with no person in common, such that the sum of the ages of the people in the first group equals the sum of the ages of the people in the second group. (Ages are given in whole numbers only.)

4. (**Bonus - 5 exam points**) Let $A_1, \ldots, A_k$ be a collection of subsets of $\{1, 2, \ldots, n\}$ such that any two have an element in common. That is, $A_i \cap A_j \neq \emptyset$ for any $i \neq j$. **(a)** Prove that $k \leq 2^{n-1}$. **(b)** Prove that there is such a collection for which $k = 2^{n-1}$.