

Math 215 - Introduction to Advanced Mathematics

Problem Set 11

Spring 2018

Due in class on Friday, May 4

For each of the following questions give your answer and then explain the reasons why your answer is correct using full sentences.

1. Prove the following using only the definition of “countably infinite (denumerable)” and the definition of what it means for two cardinalities to be equal: If A and B are both countably infinite sets, then $|A| = |B|$.
2. Prove that the set of all finite zero-one sequences (e.g. ‘010011’) is countably infinite.
3. Prove that the set of all infinite zero-one sequences S is uncountable by showing that any function from \mathbb{N} to S cannot be surjective.
4. **(3 Exam Bonus Points)** Prove that $|\mathbb{R}| = |\mathcal{P}(\mathbb{N})|$ by showing that both sets have the same cardinality as the set S from the previous problem.