Math 215 - Introduction to Advanced Mathematics

Problem Set 8

Spring 2018
Due in class on Friday, April 6

For each of the following questions give your answer and then explain the reasons why your answer is correct using full sentences.

1. A boy goes to a school that is located nine blocks east and eight blocks north of his home. Every day he walks 17 blocks to school. How many different routes are possible? (The streets follow a basic grid pattern with no diagonals or anything weird.)

2. How many routes are possible for the boy in the previous question if the block in the easterly direction that begins four blocks east and three blocks north of his home is under water and cannot be used?

3. How many ways can we pick sets A and B such that

$$\emptyset \subseteq A \subseteq B \subseteq \{1, 2, \ldots, n\}$$

where n is some positive integer?

4. Prove that

$$\sum_{k=0}^{n} \binom{m_1}{k}\binom{m_2}{n-k} = \binom{m_1+m_2}{n}$$

where m_1, m_2, and n are positive integers. **Hint**: How many ways can we choose an n-subset of a set with $m_1 + m_2$ elements? Think about how this question applies to both sides of the equation above.

5. Evaluate the expression

$$\sum_{k=0}^{n} \binom{n}{k} 2^k$$

and explain your reasoning.