Hodge modules over curves

Anh Tran

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1 Deligne's extension

Theorem 1.1. *Let $(\mathcal{V}, \nabla)$ be a flat connection on $\Delta^\times$ with quasi-unipotent monodromy. For each half interval I of length 1, $(\mathcal{V}, \nabla)$ extends uniquely to a logarithmic connection $\left(\mathcal{V}^I, \nabla\right)$ (i.e., $\nabla : \mathcal{V}^I \to \Omega_\Delta(\log 0) \otimes_{\mathcal{O}_\Delta} \mathcal{V}^I$) with all eigenvalues of the residue lying in I.*

Proof. Consider the universal cover:

$$p : \mathbb{H} \to \Delta^\times, \quad z \mapsto t = \exp(2\pi i z)$$

Since all eigenvalues of T have norm 1, they are all of the form $\exp(2\pi i\lambda)$. Picking an interval I amounts to picking a fundamental domain for p, so there exists a unique N with eigenvalues in I such that

$$T = \exp(-2\pi i N)$$

Call V a fiber of $\mathcal{V}$ (hence a fiber of $p^*\mathcal{V}$). Notice that a global section s of $\mathcal{V}$ corresponds to a $\pi_1(\Delta^\times)-$invariant section $\tilde{s}$ of $p^*\mathcal{V}$. Now, $p^*\mathcal{V}$ is trivial, i.e. $p^*\mathcal{V} \simeq V \otimes_{\mathbb{C}} \mathcal{O}_{\mathbb{H}}$ so a global section $\tilde{s}$ of $p^*\mathcal{V}$ is just $\tilde{s} : \mathbb{H} \to V$. Let $\{v_1, \ldots, v_d\}$ be a basis of V, and if we abuse notation to consider $\{v_1, \ldots, v_d\}$ as the sections (constant w.r.t. the trivialization $p^*\mathcal{V} \simeq V \otimes_{\mathbb{C}} \mathcal{O}_{\mathbb{H}}$), then we have

$$\tilde{s}_i : \mathbb{H} \to p^*\mathcal{V}, \quad z \mapsto \exp(2\pi i z N)(v_i)(z) = \sum_{j=0}^{+\infty} \frac{(2\pi i)^j z^j}{j!} N^j v_i(z)$$

which satisfies

$$\begin{aligned} \tilde{s}_i(z+1) &= \exp(2\pi i(z+1)N)v_i(z+1) \\ &= \exp(2\pi i z N)T^{-1}v_i(z+1) \\ &= \exp(2\pi i z N)v_i(z) = \tilde{s}_i(z) \end{aligned}$$

where $v_i(z+1) = Tv_i(z)$ comes from the compatibility of the $\pi_1(\Delta^\times)$-actions on $\mathbb{H}$ and $p^*\mathcal{V}$. Thus these sections descend to global sections $\{s_1, \ldots, s_d\}$ of $\mathcal{V}$, and we define the extension

$$\mathcal{V}^I \coloneqq \mathcal{O}_\Delta \cdot s_1 \oplus \cdots \oplus \mathcal{O}_\Delta \cdot s_d$$

On $\mathbb{H}$, we have the pullback connection being d since the bundle is trivial, i.e.,

$$\begin{aligned} (p^*\nabla)\tilde{s}_i &= \sum_{j=1}^{\infty} \frac{(2\pi i)^j j z^{j-1}}{j!} \mathrm{d}z \otimes N^j v_i \\ &= 2\pi i \cdot \mathrm{d}z \otimes \sum_{j=1}^{\infty} \frac{(2\pi i)^{j-1} z^{j-1}}{(j-1)!} N^j v_i \\ &= 2\pi i \cdot \mathrm{d}z \otimes N \cdot \exp(2\pi i z N)(v_i) \end{aligned}$$

so on $\Delta^\times$, noticing that $\mathrm{d}t = \mathrm{d}(e^{2\pi i z}) = 2\pi i \cdot e^{2\pi i}\mathrm{d}z = 2\pi i \cdot t\mathrm{d}z$, we have

$$\nabla s_i = \frac{\mathrm{d}t}{t} \otimes N s_i$$

so the residue of the connection is N, with eigenvalues in I as desired. ■

Definition 1.2. For simplicity of notation, we denote

$$\mathcal{V}^\alpha = \mathcal{V}^{[\alpha,\alpha+1)}, \qquad \mathcal{V}^{>\alpha} = \mathcal{V}^{(\alpha,\alpha+1]}$$

Let $j : \Delta^\times \hookrightarrow \Delta$ and $i : \{0\} \hookrightarrow \Delta$. Notice that these extensions are all subsheaves of $j_*\mathcal{V}$ (be careful, since this is the analytic category, $j_*\mathcal{O}_{\Delta^\times}$ is bigger than $\mathcal{O}_\Delta(*0)$ since meromorphic non-algebraic function can have essential singularities) since all $s_i \in H^0(\Delta^\times, j_*\mathcal{V}) = H^0(\mathbb{H}, \mathcal{V})$. Say we want to move from $\mathcal{V}^\alpha$ (with unique N_α, log of T with eigenvalues in $[\alpha, \alpha+1)$) to $\mathcal{V}^{\alpha+1}$, the new unique log of T is $N_{\alpha+1} = N_\alpha + I$. Thus

$$\tilde{s}_i^{\alpha+1} = \exp(2\pi i z)\tilde{s}_i^\alpha$$

so $s_i^{\alpha+1} = t \cdot s_i^\alpha$ since $t = e^{2\pi i z}$. It follows that $\mathcal{V}^{\alpha+1} = t \cdot \mathcal{V}^\alpha$ for all $\alpha \in \mathbb{R}$.

Note 1.1. Since $\mathcal{V}^\alpha$ is a $\mathcal{O}_\Delta$ locally free sheaf, $t \cdot \mathcal{V}^\alpha \subseteq \mathcal{V}^\alpha$, i.e., $\mathcal{V}^{\alpha+1} \subseteq \mathcal{V}^\alpha$. This is true more generally, if $\alpha \leqslant \beta$ then $\mathcal{V}^\beta \subseteq \mathcal{V}^\alpha$. ▲

Definition 1.3. We have Deligne's meromorphic extension

$$\widetilde{\mathcal{V}} = \bigcup_{\alpha \in \mathbb{R}} \mathcal{V}^\alpha \subset j_*\mathcal{V}$$

which is canonical. This is a $\mathcal{D}_\Delta$-module, where the action by ∂_t is defined via the logarithmic connection:

$$\nabla_{\partial_t} s \in \iota_{\partial_t}\left(\frac{\mathrm{d}t}{t} \otimes \mathcal{V}^\alpha\right) = \frac{1}{t}\mathcal{V}^\alpha = \mathcal{V}^{\alpha-1}$$

for $s \in \mathcal{V}^\alpha$.

Note 1.2. This name can be justified by the fact that $\widetilde{\mathcal{V}} = \mathcal{V}^\alpha \otimes_{\mathcal{O}_\Delta} \mathcal{O}_\Delta(*0)$ for any α, since $t^{-n} \cdot \mathcal{V}^{\alpha-n}$ which contains all $\mathcal{V}^\beta$ with $\beta > \alpha - n$. ▲

Note 1.3. To check that this action gives a $\mathcal{D}_\Delta$-module, we just need to check the commutator condition $[\partial_t, f]$ for $f \in \mathcal{O}_\Delta$. ▲

Proposition 1.4. $\widetilde{\mathcal{V}} = \mathcal{D}_\Delta \cdot \mathcal{V}^{-1}$, *i.e., the meromorphic extensions is generated (as a* $\mathcal{D}$*−module) by* $\mathcal{V}^{-1}$.

Proof. It suffices to show that $\partial_t \cdot \mathcal{V}^\alpha = \mathcal{V}^{\alpha-1}$ for $\alpha \leqslant -1$. Consider $s \in \mathcal{V}^{\alpha-1}$

$$\partial_t t \cdot s = \nabla_{\partial_t}(t \cdot s) = \mathrm{d}t(\partial_t) \otimes s + t\nabla_{\partial_t} s = s + t\left(\frac{\mathrm{d}t}{t}(\partial_t) \otimes N_{\alpha-1}s\right) = (N_{\alpha-1} + I)s$$

and if $\alpha \leqslant -1$ then eigenvalues of $N_{\alpha-1} + I$ are in $[-1, 0)$. It follows that it's invertible, so $\mathcal{V}^{\alpha-1} \xrightarrow{t} \mathcal{V}^\alpha \xrightarrow{\partial_t} \mathcal{V}^{\alpha-1}$ is an isomorphism, and the second map must be surjective. ■

Proposition 1.5. *The de Rham complex*

$$\mathrm{DR}\left(\widetilde{\mathcal{V}}\right) \coloneqq \left[0 \to \widetilde{\mathcal{V}} \xrightarrow{\nabla} \Omega^1_\Delta \otimes \widetilde{\mathcal{V}} \to 0\right]$$

is quasi-isomorphic to $\mathrm{R}j_*\mathbf{V}[1]$*, where* $\mathbf{V}$ *is the underlying local system of* $(\mathcal{V}, \nabla)$*.*

Proof. The idea, which we will see again in Zucker's theorem, is to filter the de Rham complex

$$V^\alpha \mathrm{DR}\left(\widetilde{\mathcal{V}}\right) = \left[0 \to \widetilde{\mathcal{V}}^\alpha \xrightarrow{\nabla} \Omega^1_\Delta \otimes \widetilde{\mathcal{V}}^{\alpha-1} \to 0\right]$$

First notice that

$$\mathrm{DR}\left(\widetilde{\mathcal{V}}\right) \simeq \left[0 \to \widetilde{\mathcal{V}} \xrightarrow{t\partial_t} \widetilde{\mathcal{V}} \to 0\right]$$

since $t : \widetilde{\mathcal{V}} \to \widetilde{\mathcal{V}}$ is invertible, and $\Omega^1_\Delta \simeq \mathcal{O}_\Delta$ (via $\Omega^1_\Delta = \mathcal{O}_\Delta \cdot \mathrm{d}t$ and ∇ is identified with ∇_{∂_t}). Thus,

$$V^\alpha \mathrm{DR}\left(\widetilde{\mathcal{V}}\right) = \left[0 \to \widetilde{\mathcal{V}}^\alpha \xrightarrow{t\partial_t} \widetilde{\mathcal{V}}^\alpha \to 0\right]$$

Next, we will show that the inclusion $V^\alpha \mathrm{DR}\left(\widetilde{\mathcal{V}}\right) \hookrightarrow \mathrm{DR}\left(\widetilde{\mathcal{V}}\right)$ is a quasi-isomorphism for all $\alpha \leqslant 0$. Since $\mathrm{DR}\left(\widetilde{\mathcal{V}}\right) = \bigcup_{\alpha \in \mathbb{R}} V^\alpha \mathrm{DR}\left(\widetilde{\mathcal{V}}\right)$, it suffices to check that the inclusion $V^\alpha \mathrm{DR}\left(\widetilde{\mathcal{V}}\right) \hookrightarrow V^\beta \mathrm{DR}\left(\widetilde{\mathcal{V}}\right)$ is a quasi-isomorphism for $\beta < \alpha \leqslant 0$. This is equivalent to the quotient complex

$$\left[0 \to \widetilde{\mathcal{V}}^\beta / \widetilde{\mathcal{V}}^\alpha \xrightarrow{t\partial_t} \widetilde{\mathcal{V}}^\beta / \widetilde{\mathcal{V}}^\alpha \to 0\right]$$

being quasi-isomorphic to 0. The key point here is that the $V-$filtration is discrete (here it's because the set of eigenvalues - exactly where the filtration jumps - is discrete), so $\widetilde{\mathcal{V}}^{\beta+\epsilon} = \widetilde{\mathcal{V}}^{>\beta}$ for all ϵ small enough. In other words, the above complex is quasi-isomorphic to

$$\left[0 \to \widetilde{\mathcal{V}}^\beta / \widetilde{\mathcal{V}}^{>\beta} \xrightarrow{t\partial_t} \widetilde{\mathcal{V}}^\beta / \widetilde{\mathcal{V}}^{>\beta} \to 0\right] = \left[0 \to \mathrm{gr}^\beta_V \widetilde{V} \xrightarrow{t\partial_t} \mathrm{gr}^\beta_V \widetilde{V} \to 0\right]$$

but $\mathrm{gr}^\beta_V \widetilde{V}$ is the generalized eigenspace of $t\partial_t$ so $t\partial_t$ is invertible ($t\partial_t - \beta$ is nilpotent, and $\beta < 0$ so eigenvalues are nonzero). In particular,

$$\mathrm{DR}\left(\widetilde{\mathcal{V}}\right) \simeq \left[0 \to \widetilde{\mathcal{V}}^0 \xrightarrow{t\partial_t} \widetilde{\mathcal{V}}^0 \to 0\right]$$

and then we can show that the germ at 0 of this complex is quasi-isomorphic to

$$\left[0 \to \mathrm{gr}^0_V \widetilde{V} \xrightarrow{t\partial_t} \mathrm{gr}^0_V \widetilde{V} \to 0\right]$$

which is equivalent to showing

$$\left[0 \to \widetilde{\mathcal{V}}^{>0} \xrightarrow{t\partial_t} \widetilde{\mathcal{V}}^{>0} \to 0\right] \simeq 0$$

which we can check explicitly. ■

Note 1.4. Another (possible way) of seeing this is by showing (by looking at stalks) that

$$\mathcal{H}^0\left(\mathrm{DR}\left(\widetilde{\mathcal{V}}\right)\right) \simeq \mathrm{R}^1 j_*\mathbf{V}, \quad \mathcal{H}^{-1}\left(\mathrm{DR}\left(\widetilde{\mathcal{V}}\right)\right) \simeq j_*\mathbf{V}$$

and since we are on Δ, the Ext^2 and above vanishes thus complex splits as direct sum of its cohomologies. ▲

Now let's look at
$$\mathcal{M} \coloneqq \mathcal{D}_\Delta \cdot \mathcal{V}^{>-1} \subseteq \widetilde{\mathcal{V}}$$
(we don't necessarily have equality, since $N_{>-2} + I$ can have 0 as an eigenvalue hence not invertible). In this case we have
$$\mathrm{DR}(\mathcal{M}) = j_*\mathbf{V}[1]$$
which is the minimal extension of $\mathbf{V}$ (so $\mathcal{M}$ should be the $\mathcal{D}-$module minimal extension of $\mathcal{V}$). Now we can define the V-filtration on $\mathcal{M} : V^\alpha\mathcal{M} = \mathcal{V}^\alpha \cap \mathcal{M}$.

Note 1.5. We have that $V^\alpha\mathcal{M} = \mathcal{V}^\alpha$ for $\alpha > -1$. Furthermore,
$$t \cdot V^\alpha\mathcal{M} = \mathcal{V}^{\alpha+1}, \quad \partial_t \cdot V^\alpha\mathcal{M} = V^{\alpha-1}\mathcal{M} \quad \alpha \leqslant -1$$
and we can identify $\mathrm{Gr}_V^\alpha \coloneqq V^\alpha\mathcal{M}/V^{>\alpha}\mathcal{M}$ with the generalized $\alpha-$eigenspace of $t\partial_t$ as follows: we have
$$\mathcal{V}^\alpha/\mathcal{V}^{\alpha+1} = \mathcal{V}^\alpha/t \cdot \mathcal{V}^\alpha \simeq \mathcal{V}^\alpha \otimes \mathbb{C}\{t\}/(t) = \mathcal{V}^\alpha\big|_0$$
and at the origin, $t\partial_t \cdot s = N_\alpha s$. Thus $\mathcal{V}^\alpha/\mathcal{V}^{>\alpha}$ is the generalized eigenspace (we don't have an actual eigenspace since technically N splits into a semisimple part and a nilpotent part). ▲

2 Hodge modules over a curve

Recall that we have a category $\mathrm{MF}_{\mathrm{rh}}(\mathcal{D}_X, \mathbb{Q})$ whose objects consist of

- A perserve sheaf $\mathbf{V}$ over $\mathbb{Q}$;
- A regular holonomic $\mathcal{D}_X$-module $\mathcal{M}$ with an isomorphism $\mathrm{DR}(\mathcal{M}) \simeq \mathbf{V} \otimes \mathbb{C}$;
- A good filtration $F_\bullet$ on $\mathcal{M}$.

and our desired category of Hodge modules will be a subcategory of $\mathrm{MF}_{\mathrm{rh}}(\mathcal{D}_X, \mathbb{Q})$. There will be two types of building blocks:

- Given an inclusion $i : \{x\} \hookrightarrow X$ and a polarizable pure Hodge structure $(H_\mathbb{Q}, F^\bullet)$ of weight w, our Hodge module of type 0 and weight w will be $i_+H_\mathbb{C}$ with the filtration induced by $F^\bullet$, and the perverse sheaf being the skyscaper sheaf $H_\mathbb{Q}, x$.
- Given a polarizable variation of Hodge structure $(\mathbf{V}_\mathbb{Q}, F^\bullet)$ of weight $w - 1$ over a Zariski open subset $j : U \hookrightarrow X$, our Hodge module of type 1 and weight w will be the minimal extension (as a $\mathcal{D}-$module) of $\mathcal{V} = \mathbf{V}_\mathbb{Q} \otimes_\mathbb{Q} \mathcal{O}_U$, with the underlying perverse sheaf $j_*\mathbf{V}_\mathbb{Q}[1]$, and a filtration as below.

It suffices to do this for punctured disc. Now let's assume that $(\mathcal{V}, \nabla, \mathbf{V}_\mathbb{Q})$ is a polarized variation of Hodge structures of weight $w - 1$ over $\Delta^\times$. Denote
$$F_p\mathcal{V} \coloneqq F^{-p}\mathcal{V}$$
the Hodge filtration on $\mathcal{V}$. We have $\partial_t \cdot F_p\mathcal{V} \subseteq F_{p+1}\mathcal{V}$ by Griffiths' transversality. The naive choice $F_p\mathcal{M} \coloneqq \mathcal{M} \cap j_*F_p\mathcal{V}$ is bad since for $p \gg 0$ we have $F_p\mathcal{V} = \mathcal{V}$ so $F_p\mathcal{M} = \mathcal{M}$ but this is not $\mathcal{O}_\Delta$-coherent. By Schmid's nilpotent orbit theorem, the subbundles $F_\bullet\mathcal{V}$ extends to subbundles of the Deligne's extensions:
$$F_\bullet\mathcal{V}^\alpha, \quad F_\bullet\mathcal{V}^{>\alpha}$$
and we can actually show that $F_p\mathcal{V}^\alpha = \mathcal{V}^\alpha \cap j_*F_p\mathcal{V}$ and similarly for $F_p\mathcal{V}^{>\alpha}$.

Note 2.1. $F_p\mathcal{V}^\alpha$ is a subbundle of $\mathcal{V}^\alpha$, so $\mathcal{V}^\alpha/F_p\mathcal{V}^\alpha$ is also a vector bundle (quotient by subbundle is fine). A local section $\sigma \in \mathcal{V}^\alpha \cap j_*F_p\mathcal{V}$ on $U \ni 0$ is a section of $\mathcal{V}^\alpha$ over U and a section of $F_p\mathcal{V}$ over $U - \{0\}$. Then the image (under quotient) $\overline{\sigma}$ is a local section, over U of $\mathcal{V}^\alpha/F_p\mathcal{V}^\alpha$. It's 0 on $U - \{0\}$, but $\mathcal{V}^\alpha/F_p\mathcal{V}^\alpha$ is locally free so $\overline{\sigma} = 0$ on U thus $\sigma \in F_p\mathcal{V}^\alpha$. ▲

In order to get Griffths transversality (which is necessary to be a good filtration) we define

$$F_p\mathcal{M} = \sum_{j=0}^{+\infty} (\nabla_{\partial_t})^j F_{p-j}\mathcal{V}^{>-1}$$

which is a finite sum since $F_p\mathcal{V} = 0$ for $p \ll 0$. It's exhaustive since $F_p\mathcal{V}^{>-1} = \mathcal{V}^{>-1}$ for $p \gg 0$, and $\mathcal{M} = \mathcal{D}_\Delta \cdot \mathcal{V}^{>-1}$. We also have

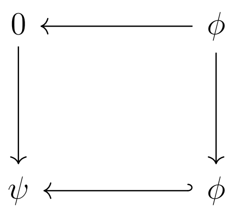
$$\partial_t \cdot F_p\mathcal{M} = F_{p+1}\mathcal{M}, \quad p \gg 0$$

so $F_\bullet\mathcal{M}$ is a good filtration. $(\mathcal{M}, F_\bullet\mathcal{M}, j_*\mathbf{V}[1])$ is our Hodge module of type 1. Let $\mathrm{MH}(\Delta, w)^p$ be the full subcategory of $\mathrm{MF}_{\mathrm{rh}}(\mathcal{D}_\Delta, \mathbb{Q})$, consisting of finite direct sums of objects of these two types.

Theorem 2.1. $\mathrm{MH}(\Delta, w)^p$ *is abelian and semisimple.*

Proof. We first show that there are no nonzero morphisms between objects of different types. We know that the underlying perserve sheaves of type 0 and 1 correspond to quivers

A morphism of hodge modules would correspond to a morphism of quivers. But this is impossible. For example suppose we have linear maps $0 \to \psi$ and $\phi \to \phi$, then



must commute, but this forces $\phi \to \phi$ to be zero, so the morphism of quivers is 0. For same type statements, we have that the category of polarizable (variations of) Hodge structures is both abelian and semisimple. ■

Theorem 2.2 (Zucker)**.** *The cohomology* $H^i(\Delta, j_*\mathbf{V}_\mathbb{C})$ *carries a polarized Hodge structure of weight* $w + i$.

Very rough sketch. We don't have a monodromy filtration on the nose, since our T is not unipotent (which gives nilpotent residue), but we do have

$$\mathrm{Gr}_V^\alpha\mathcal{M} = \psi_t^\alpha\mathcal{M}$$

which is a generalized α-eigenspace, so we can lift the nilpotent filtration to $N_\bullet V^\alpha\mathcal{M}$. We have a $V-$filtration on $\mathrm{DR}(\mathcal{M})$ by setting

$$V^\alpha\mathrm{DR}(\mathcal{M}) := \left[0 \to V^\alpha\mathcal{M} \xrightarrow{\nabla} \Omega_\Delta^1 \otimes V^{\alpha-1}\mathcal{M} \to 0\right]$$

For any $\alpha < 0$, the induced morphism

$$\mathrm{Gr}_V^\alpha \mathcal{M} \xrightarrow{\partial_t} \mathrm{Gr}_V^{\alpha-1}\mathcal{M}$$

is an isomorphism, so $V^0\mathrm{DR}(\mathcal{M}) \hookrightarrow \mathrm{DR}(\mathcal{M})$ is a quasi-isomorphism. Since $\mathcal{M}$ is a minimal extension, we actually have that $\mathrm{DR}(\mathcal{M})$ is quasi-isomorphic to

$$\left[0 \to N_0 V^0\mathcal{M} \xrightarrow{t\partial_t} N_{-2}V^0\mathcal{M} \to 0\right]$$

On the other hand we have the L^2-de Rham complex

$$\mathrm{DR}\left(\widetilde{\mathcal{V}}\right)_{(2)} = \left[0 \to \widetilde{\mathcal{V}}_{(2)} \xrightarrow{\nabla} \left(\Omega^1 \otimes \widetilde{\mathcal{V}}\right)_{(2)} \to 0\right]$$

and Schmid's characterization of $N_\bullet \mathcal{V}^\alpha$ gives that

$$\left(\Omega^1 \otimes \widetilde{\mathcal{V}}\right)_{(2)} = \frac{\mathrm{d}t}{t} \otimes N_{-2}\mathcal{V}^0, \quad \widetilde{\mathcal{V}}_{(2)} = N_0\mathcal{V}^0$$

so we get

$$\mathrm{DR}\left(\widetilde{\mathcal{V}}\right)_{(2)} \simeq \mathrm{DR}(\mathcal{M}) = j_*\mathbf{V}_\mathbb{C}$$

Now, let $\mathcal{H} = \mathcal{C}^\infty_{\Delta^\times} \otimes \mathbf{V}$. It has a flat C^∞-connection $D = D' + D''$ where $D'' = d'' \otimes \mathrm{Id}$ and D' induced by ∇. One can then define the L^2-de Rham complex $\mathcal{L}^\bullet_{(2)}(\mathcal{H}, D)$. The inclusion $\mathrm{DR}\left(\widetilde{\mathcal{V}}\right)_{(2)} \hookrightarrow \mathcal{L}^\bullet_{(2)}(\mathcal{H}, D)$ is a quasi-isomorphism, so we get our decomposition. ■

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