

UNDERGRADUATE



MAJOR IN THE

MATHEMATICAL

SCIENCES

A REPORT OF CUPM

THE COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS



THE MATHEMATICAL ASSOCIATION OF AMERICA

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The Undergraduate Major in the Mathematical Sciences

Introduction

As the 21st century approaches, better mathematics preparation and involvement of more students in higher levels of achievement are recognized as central components in efforts to remain competitive in the world economy. The collegiate mathematics community, aware of the foundational nature of mathematics as a driving force behind technological change, is giving serious attention to these concerns. (See, in particular, [6, 14, 15, 32, 33, 42, 48, 52].) Although some problems are beyond the scope of college faculty to address directly [20], mathematicians can develop and implement a college mathematics curriculum that meets current needs and gives cogent attention to methodologies for teaching mathematics.

During the 1950s, the Mathematical Association of America (MAA) organized its Committee on the Undergraduate Program in Mathematics (CUPM) to express and shape consensus in the mathematical community concerning the undergraduate curriculum. CUPM recommendations about the undergraduate mathematics major appeared in 1965 [17], 1972 [12], and 1981 [54]. The CUPM Subcommittee on the Undergraduate Major (SUM) was created in 1987 to update the 1981 publication. After a careful review of the 1981 report and its relationship to previous reports, SUM solicited views from broad sectors of the mathematics community, placed calls in national publications [8, 50], held public hearings, exchanged information with committees having related charges, and encouraged dialogue both at its meetings and by correspondence. The Subcommittee's report was adopted by CUPM at its January 1991 meeting.

The curriculum recommendations that follow present both a philosophy to guide curriculum development and a framework on which to

construct departmental requirements for the undergraduate mathematical sciences major. Issues of teaching and advising are natural adjuncts to the discussion. The program structure takes into account the realities of current practice and reflects successful experience. This common framework provides the opportunity for choices among a broad variety of courses while assuring both depth and flexibility. This report is offered in a helpful spirit as a practical tool for those who want to provide strong preparation for students having widely varying goals. By issuing these recommendations at this time of curricular revitalization at all levels in mathematics education, CUPM hopes to help sustain the momentum for reform in undergraduate mathematics and to provide answers for those who want assistance in their efforts to bring about effective change.

Philosophy

Students who complete mathematics majors have often been viewed by industry, government, and academia as being well-prepared for jobs that require problem solving and creative thinking abilities. The philosophical views that underlie the concrete recommendations of this report provide a basis to ensure that this reputation is upheld and enhanced. The first five tenets in this philosophy reflect timely small changes from those in the 1981 CUPM report [54]; the added four reflect new realities and national issues.

- I. Attitudes and skills.** The mathematics curriculum should have as a primary goal developing the attitudes of mind and analytical skills required for efficient use, appreciation, and understanding of mathematics. It should also focus on the student's ability to function as an independent mathematical learner. The development of rigorous mathematical reasoning and of abstraction from the particular to the general are two themes that should run throughout the curriculum.
- II. Program level.** The mathematical sciences curriculum should be designed around the abilities and academic needs of the typical mathematical sciences student while making supplementary material or courses available to attract and challenge particularly talented students.
- III. Interaction.** Since active participation is essential to learning mathematics, instruction in mathematics should be an interactive process in which students participate in the development of new concepts,

questions, and answers. Students should be asked to explain their ideas both by writing and by speaking, and should be given experience working on team projects. In consequence, curriculum planners must act to assure appropriate sizes of various classes. Moreover, as new information about learning styles among mathematics students emerges, care should be taken to respond by suitably altering teaching styles.

- IV. Applications and theory.** Applications should be used wherever appropriate to motivate and illustrate material in abstract and applied courses. The development of most topics should involve an interplay of substantive applications, mathematical problem solving, and theory. Theory should be seen as useful and enlightening for all courses in the mathematical sciences, regardless of whether they are viewed as pure mathematics or applied mathematics.
- V. Recruiting.** First courses in a subject should be designed to appeal to as broad an audience as is academically reasonable so that students entering college will be attracted to the discipline by the vitality of subjects at the calculus level. Students in such courses should be led to sense the intellectual vitality of the mathematical approach to problems and of the construction of theories that resolve those problems.
- VI. Concentrations.** A system of tracks within the mathematical sciences major is an appropriate response to the diverse interests of students and to the potential opportunities for undergraduate majors. However, the undergraduate mathematics program, and the courses in it, should be designed in such a way that students are not forced to make choices about their post-baccalaureate lives too early in their undergraduate careers. In particular, all tracks should incorporate the major intellectual components of the discipline—including the interplay of theory and applications, study in depth, and the construction of general theories and proofs—so that bachelor's graduates will retain maximum flexibility in pursuing diverse opportunities for employment or further study.
- VII. Technology.** Computers have transformed the world in which today's undergraduates live and work. Recent advances have made it possible for computers and graphing calculators to play an important and expanding role in the teaching and learning of mathematics, just as they do in mathematical research. Curriculum planners

should take advantage of modern computing technology. Beyond being competent in a high-level programming language, mathematical sciences majors should be familiar with symbolic manipulation software. In applied courses such as statistics, numerical analysis and operations research, they should be exposed to standard software packages used by practitioners.

VIII. Transitions. The nurturing of the next generation of doctoral mathematical scientists is a special shared responsibility of all undergraduate and graduate teachers and advisors. College and university faculties should work together to ease the transition for students who move from mathematical sciences majors to the more theoretical focus of study for the master's or doctoral degrees. Professional society channels should be used as a resource both for communications and for recommendations concerning graduate study.

IX. Advising. Advising of mathematics majors throughout their undergraduate years is an important responsibility. Careful and sustained individualized advising is a crucial aspect of the student's mathematics program. Creating specialized concentrations within a department's major imposes added advising responsibilities.

The Program

This section describes a general intellectual and support framework for the mathematical sciences major of the 1990s which is designed to implement the tenets of the philosophy enumerated above. The framework is shaped by a curricular *structure* which has fixed *components* but allows considerable latitude in the choice of specific courses. Combined with the creation of specialized curriculum concentrations or *tracks* within the major, this structure provides both flexibility and utility. It does, however, impose a serious advising responsibility on the mathematics faculty. Moreover, since the effectiveness of any curriculum depends strongly on the quality of teaching, mathematicians will need to become aware of research, innovations, and recommendations concerning undergraduate teaching. (See, for example, *Undergraduate Mathematics Education Trends* and many of the references cited below, including [6, 7, 16, 19, 29, 32, 33, 39, 41, 42, 43, 45, 47, 49, 51, 57, 58].)

Advising

Unlike an earlier, simpler day when all mathematics majors took essentially the same sequence of courses with only a few electives in the senior year, the typical undergraduate mathematical sciences department today requires students to make substantial curricular choices. As a result, departments have advising responsibilities of a new order of magnitude. Students need departmental advice as soon as they show interest in (or potential for) a mathematics major. Advisors should carefully monitor each advisee's academic progress and changing goals, and together they should explore the many intellectual and career options available to mathematics majors. Career information (e.g., [28]) that informs students of the wide variety of career options open to them is important. If a "minor" in another discipline is a degree requirement or option, then achieving the best choice of courses for a student may necessitate coordination between the major advisor and faculty in another department.

Advisors should pay particular attention to the need to retain capable undergraduates in the mathematical sciences pipeline, with special emphasis on the needs of under-represented groups [23, 34, 37, 39, 52, 53]. When a department offers a choice of several mathematical tracks within the major, advisors have the added responsibility of providing students with complete information even when students do not ask many questions. Track systems may lead students to make life-time choices with only minimal knowledge of the ramifications; therefore, departments utilizing these systems for their majors must assure careful and timely information. One requisite of an individualized approach to advising is that each advisor be assigned a reasonable number of advisees. (See also the chapter on advising in [42].)

Structural Components

This section describes a framework which is a useful tool for constructing course requirements for a department's mathematical sciences major. This structure involves both specific courses (e.g., "linear algebra"), and more general experiences (e.g., "sequential learning") derived through those courses.

Seven components form the structure for a mathematical sciences major:

- A. Calculus (with Differential Equations)
- B. Linear Algebra

- C. Probability and Statistics
- D. Proof-Based Courses
- E. An In-Depth Experience
- F. Applications and Connections
- G. Track Courses, Departmental Requirements, and Electives.

The first six components will normally require nine or ten courses, at least seven of which would be taught by the major department. (By a course, we mean a three or four semester-hour course.) Local curricular requirements or the desire to achieve strong preparation may necessitate further specificity in the major. Choices within components should be based on close consultation between student and advisor. To facilitate coherent choices, a department may organize a system of specialized concentrations within the major. Additional courses appropriate to particular tracks are part of the last of the structural components.

Under normal circumstances, every track of a department's mathematical sciences major should require courses fitting each of these seven components. Exceptions would be warranted in those situations where a track offered by a mathematical sciences department conforms to the curricular recommendations of another professional society—for example, a computer science track conforming to current national guidelines (presently [55]) or a statistics track following American Statistical Association (ASA) recommendations. Departments should name such special tracks appropriately, and should inform students of the sources of the curricular recommendations that are the basis for any special tracks.

A. Calculus (with Differential Equations)

All mathematical science major programs should include the content of three semesters of calculus. This requirement is central to the major. There is much interest in the mathematical community in reworking calculus courses, originating during the late 1980s [16, 46] and continuing into the 1990s [20, 56]. Several professional societies and funding agencies have focused their attention on calculus, so the resulting prototypes should receive consideration during curriculum planning. Since more client disciplines require calculus than any other course in the major, serious attention to concerns of those disciplines is necessary and appropriate. For example, the most recent recommendations for computer science students include a minimum of two calculus courses which are to include series and differential equations. These recommendations, which were in preparation during the period 1988-1990, also place heavy

emphasis on theory, definition, axioms, theorems and proof as a "process . . . present in . . . introductory mathematics courses—calculus or discrete mathematics." [55]

Calculus courses show wide variation in arrangement of topics, levels of rigor, and methods of presentation. Central topics from calculus normally include discussion of limits, derivatives, integrals, the Fundamental Theorem of Calculus, transcendental functions, elementary differential equations, Taylor's theorem, series, curves and surfaces, partial differentiation, multiple integration, vector analysis including Green's, Stokes', and the divergence theorems, and experience with approximations, numerical methods, and challenging computations. Better-prepared students may be given much more, and the level of rigor and sophistication will vary substantially from course to course.

This requirement will typically be satisfied by the equivalent of a sequence of three mathematics courses.

B. Linear Algebra

Every mathematical science major should include a course in linear algebra. This requirement serves three distinct purposes. First, it introduces students to a part of mathematics which is the foundation of an ever-widening family of significant applications. Second, it provides a geometrically based preparation for many upper-level courses in algebra and analysis. Third, it serves as a needed bridge between lower- and upper-division mathematics courses. Since first-year calculus is usually taught as a service course with little emphasis on theory, many mathematicians see a balancing emphasis on mathematical structure with an introduction to theorems and proofs as an essential and natural component of linear algebra. (When linear algebra is an integral part of the lower-division component of the major, close consultation between the two- and four-year colleges in a given region is advisable.)

The content of the first linear algebra course is currently under discussion in the mathematical community. The traditional vector-space-oriented linear algebra course begins with matrix calculations, develops vector space ideas starting with R^2 , moves to R^n , and then to the study of abstract vector spaces and their linear transformations; it culminates in a brief treatment of eigenvalues and eigenvectors. A second type of course is matrix-based: it emphasizes more applications and computational topics such as LU decomposition and pseudoinverses, and stresses the pervasive use of linear modeling in quantitative disciplines. A com-

positive approach which is receiving increased attention [5] gives the linear algebra course a more matrix-based focus while maintaining the same level of mathematical rigor as the traditional vector space approach.

Actual applications of linear algebra are rarely small and neat. As a result, many linear algebra software packages have been developed and are widely used by practitioners. How these packages should be used in the introductory courses is not yet well established; however, there should be active experimentation on the use of these mathematical tools in the first linear algebra course.

This requirement will generally be satisfied with the equivalent of one mathematics course, although the essentials of linear algebra may be included in some calculus sequences.

C. Probability and Statistics

Every mathematical sciences major should include at least one semester of study of probability and statistics at a level which uses calculus as a prerequisite. Many elements of the chapter on statistics in the 1981 CUPM report [54] are still timely; see also [11, 29].

Since it is a severe limitation to compress an introduction to probability and an understanding of statistics into one course, care must be taken that it not be a course focusing only on statistical theory. Versions of this course can be developed in which about a month of a suggested fourteen-week syllabus is devoted to probability theory; other courses in the curriculum (e.g., discrete methods, modeling, operations research) can be coordinated to include additional basic probability topics [24, 4].

The major focus of this course should be on data and on the skills and mathematical tools motivated by problems of collecting and analyzing data. Data collection and description are important. It meets practical needs and helps motivate the difficult idea of sampling distribution [29]. Students must be trained to look at data and be aware of pitfalls to see for themselves the results of repeated random sampling and the variability of data [4, 10, 18, 30, 40, 51].

The probability and statistics course description in the 1981 CUPM report included a computer programming prerequisite and a recommendation that instructors "use library routines or pre-written programs if they wish." Because statistical software packages have become so important in the actual uses of statistics in recent years, any statistics course taught now should use a nationally available software package.

This requirement will generally be satisfied with the equivalent of one mathematics course.

D. Proof-Based Courses

All students in a mathematics or mathematical sciences major should have within their program two terms of proof-based mathematics with a calculus-level prerequisite. One of these courses should be in algebra (or discrete mathematical structures) and one in analysis (continuous mathematics).

Such courses can come in a myriad of attractive packages, and it is not the intent of these recommendations to specify categorically which particular courses would be good selections for the various tracks. However, all students should encounter sustained mathematical discussion that involves the concatenation of definitions and theorems to build a substantial mathematical structure. Indeed, such an intellectual configuration has been one of the central paradigms within mathematics during this century, and all mathematics students should be exposed to it, regardless of where their majors will ultimately lead.

Departments will find that this component in the major structure can be filled in quite different ways. Two natural choices within this category are a mature course in modern algebra and a proof-based course in analysis which could be any of what are often entitled real variables, advanced calculus, introduction to functional analysis, or complex analysis. (Departments should strongly recommend completion of both the traditional algebra and analysis undergraduate sequences for students intending to pursue graduate work in a mathematical science.) Some departments offer a very rigorous introduction to linear algebra, and such a course could be used as the algebra part of this component. Additionally, other courses, different from the traditional ones mentioned above could fulfill this component. For example, courses in mathematical logic, computational complexity, linear programming, and graph theory can be taught in ways that would make them appropriate.

This requirement will generally be satisfied with the equivalent of two mathematics courses, although a rigorous course used for the linear algebra component might also satisfy the algebra part of this requirement.

E. An In-Depth Experience

Every mathematical sciences major should pursue in depth at least one mathematical area beyond calculus. The aim of this component in the major structure is that the student participate in a sustained two-course sequence in at least one important area of mathematics. For many

students a requirement like this is best met by a two-course sequence with a calculus prerequisite such as the traditional abstract algebra-real analysis sequences. In addition to any second course that builds on one of the proof-based courses outlined above, a variety of other combinations would be useful ways to assure this in-depth mathematical experience for some students. Examples include a two-term sequence in probability and statistics, numerical analysis, or combinatorics and graph theory. The critical condition is that there be a coherent sequence with the second course using and building on the first which in turn has a calculus-level prerequisite.

There are many ways to satisfy this requirement with the equivalent of one mathematics course beyond those required to satisfy the four previous components; some choices (e.g., numerical analysis) may involve two additional mathematics courses.

F. Applications and Connections

All mathematics and mathematical sciences major programs should include two types of courses that lie outside the core of mathematics. Both of these courses should emphasize the applications of mathematics and its connectedness to other disciplines. Although many options can be identified for the two courses in this component to meet individual student goals, most students would be served best by one course of each of the following two types:

A course using computers. For a student with minimum computer experience, the best option would be a computer science course that introduces well-structured programming and algorithmic problem solving in the context of a modern programming language, and includes such topics as an introduction to data structures and the analysis of algorithmic complexity. (The standard Computer Science I course such as described in [55] satisfies this requirement.) For a student entering the major with adequate capability in a high-level programming language this requirement may be fulfilled with a more advanced computer science course or a quantitative course in the mathematical sciences which requires the use of computers to solve pertinent problems. Examples include a course in operations research, linear or nonlinear programming, control theory, statistics, applied mathematics, or a mathematical modeling course. (Such courses may, depending on the institution's structure be taught either within or outside of the major department; see also [45-58].)

A course applying mathematical methods. Typically, this will be a quantitative course outside the mathematical sciences that makes substantial use of the material in one or more of the mathematics courses that are part of the previous components. The intent of this component is to give students a taste of how mathematics is applied in another discipline and is meant to go well beyond the examples typically seen in mathematics courses. This course must have significant mathematical usage and be taught outside the mathematical sciences; examples include calculus-based physics, mathematical economics, and mathematical biology.

This requirement involves two courses normally taught in departments outside the mathematical sciences.

G. Track Courses, Departmental Requirements, and Electives

The major will be completed by courses from three other categories. Usually there will be certain designated courses (or a choice among such courses) that are essential for a particular track; some of these may be taught outside the mathematical sciences and, depending on institutional constraints, may or may not be counted as credit in the major. In addition, there may be institution-wide or departmental requirements such as senior projects, seminars, or other broadening experiences that are offered as courses. There may also be undesignated electives, some number of which may be required to complete the major. This is especially the case when a student skips early courses (perhaps through the Advanced Placement Program) or when there are higher than usual credit requirements for the major. Additional courses of particular relevance to the student's goals should be selected with careful faculty advice.

The number of courses for this component will vary by department and track.

The total number of courses needed to satisfy components A through F is nine or ten, but one or two of these are not normally taught in a mathematical sciences department. A student who begins with Calculus I will typically satisfy A through F with eight mathematical science courses, some of which may be equivalent to four semester hours in length. Component G will typically include from one to four additional courses, some of which may not be in the major department.

Curriculum Tracks

All tracks (concentrations of courses) in a department's mathematical sciences major should begin with a set of common courses; students are not normally sufficiently informed to choose a track wisely until their junior year. Although tracks give the major a particular focus, it can be argued that general mathematical techniques and reasoning skills are, in fact, the primary asset of mathematics graduates as they begin initial jobs or move to graduate study. It may further be argued that early concentration is not as important as some students and faculty appear to believe.

Many colleges offer several formalized clusters of courses that may be chosen by students majoring in the mathematical sciences; these tracks may be offered in one or several mathematical sciences departments. Typically a department will have two or more designated tracks with some choices of courses within each track. Particular attention must be given to the needs of students who initially choose one track and then decide to change, to students lacking familiarity with the academic and technical worlds who may need extra help and time in making choices, and to retention of students from under-represented groups.

Appropriate choices of courses for each of the structural components of the major will be departmentally designated or worked out in consultation with faculty advisors, taking into account employment and graduate school constraints. The strong basis provided by structural components A through F gives the opportunity to design meaningful coherent clusters of track courses in structural component G. Departments should adhere with tenacity to the component structure as a basis for tracks within the major except when, as noted above, the track follows recommendations developed by another appropriate professional society. Departments in the mathematical sciences should make their faculty and students aware of applicable guidelines related to the tracks offered their students.

The names of tracks vary considerably from one institution to another. For example, some schools label a track much like the one described below under the title "Computational and Applied Analysis" as "Applied Mathematics" or "Engineering Support." Most of the tracks may be broadly considered as options within applied mathematics. Every applied mathematics track should include some fundamental components of "pure" mathematics just as the pure mathematics student should gain some understanding of applied and computational mathematics.

For some tracks, considerable specificity for components A through F

may be desirable; for others, a large number of courses may be needed in the specialized G component. As a consequence, some tracks may be unattractive in a setting where there are institutional limitations on the total number of hours which may be required in the major. Here are some examples of tracks that are commonly found in departments of mathematical sciences:

Actuarial Mathematics Strong programs include differential equations and courses in linear algebra, advanced calculus, computer science and mathematical economics; these courses easily fit in the framework of components A through F. Basic courses in accounting and numerical analysis are desirable, as are specialized courses in actuarial or risk theory and life contingencies. Departments offering this track will want to keep abreast of the content of actuarial tests and other information available from the actuarial societies. (See, for example, materials of the Casualty Actuarial Society [9], the Society of Actuaries [46], and the journal entitled *The Student Actuary* [2].)

Applied Statistics For applied statistics, the structural components A through G usually include analysis, numerical analysis, and discrete mathematics. A balance of theory, methods, and applications courses in statistics may include, at the upper-division level, topics such as experiment design and analysis, regression analysis, quality control, sampling methods, data analysis and multivariate methods, nonparametric statistics, and elective topics. Utilization of statistical software must be emphasized. Courses from the Management Science track are helpful. Courses in Computer Science such as advanced programming techniques (including elementary data structures), operating systems, and computability are also appropriate.

Num. Anal. *Computational and Applied Analysis* Within A through F, students should include courses from differential equations, advanced calculus or real analysis, complex analysis, and numerical analysis. Modeling and simulation courses are also important. Strong work in the physical sciences is advisable. Somewhere in the course structure project or team-oriented assignments should be emphasized.

Computer Analysis A track labeled Computer Science should follow the recommendations of the professional computer societies [55]. Even if offered in a joint or mathematical sciences department, such majors are computer science majors. In a small department, a mathematical sciences major which includes a number of computer science courses but

does not meet fully the requirements of [55] might also include a year of rigorous discrete mathematics and additional courses from operations research, modeling and simulation, abstract algebra or discrete structures, combinatorics, and applications areas.

Stat.

Management Science Students desiring preparation in management science should select their mathematical electives from the following courses: advanced statistics, decision mathematics, design and analysis of experiments, applied regression, operations research, modeling and simulation, and mathematical programming. In addition, students should consider taking courses in accounting, economics, business law, and principles of management or marketing.

Operations Research Beyond calculus and linear algebra, recommendations for this track include advanced calculus, two courses in probability and statistics, linear programming, stochastic models, combinatorics or graph theory, and two courses in computer science (the second a course with a substantial amount of data structures). There should also be at least one course in economics. A track with courses very similar to those for Operations Research may be labeled Discrete Mathematics; the two tracks sometimes exist in the same department. Coding theory is included in Discrete Mathematics, and there is often a heavier emphasis on theoretical computer science.

Pure Mathematics This option is sometimes labeled Core Mathematics and might also be termed Classical Mathematics. One year each of abstract algebra and real analysis should be included; this requires one course more than the minimum number to satisfy components B and D. Strong additions are courses in topology or geometry, mathematical logic, complex variables, number theory; some numerical analysis or theoretical computer science is advised.

Simulation

Scientific Computing A modern thrust in science and mathematics is the use of computers to cast light on disciplinary issues. This has led to a new subject area within the mathematical sciences in which students study computational and graphical techniques with an eye to their application in other scientific or mathematical subject areas. Within this option, typical mathematics courses after calculus and linear algebra include ordinary, partial and numerical differential equations, optimization, and modeling. Modern programming languages, data structures, parallel and vector processing, computer graphics, computer simulation, and software engineering are important courses from computer science, as is

familiarity with a variety of important computational software packages.

Systems Analysis Students with general interest in systems analysis or information science should choose courses from the Management Science track. In addition, they should consider courses in accounting, application programming, advanced programming techniques, software engineering, databases and file structures, and management information and decision support systems. This track should emphasize project-oriented assignments.

Teaching (Secondary) After the full calculus sequence, real analysis (or advanced calculus) should be included in this track because graduates need this preparation to teach strong calculus courses in the high school curriculum. In addition, courses in Euclidean and non-Euclidean geometry, abstract algebra, modeling, history of mathematics, and number theory are very useful for the prospective secondary school teacher. (More specific information is contained in the COMET and NCTM guidelines [25, 31].)

Completing the Major

The mathematical sciences major should also involve a variety of other types of experiences and activities that are, in some cases, "co-curricular." Most of the following program elements will help mathematical sciences graduates communicate with colleagues, work with groups of people with differing backgrounds, and build their mathematical self-confidence. (See also [2, 3, 19, 33, 39, 41, 43, 48, 49, 53, 57].)

Integrative Experiences

Every student who majors in mathematics should be encouraged to think about the discipline as a whole. The distinctive nature of mathematical activity, its history and place in the intellectual realm, and its usefulness in society at large are matters on which majors should reflect. This aim may be addressed in a variety of different ways by departments and institutions. The form such an activity assumes is highly dependent on departmental faculty and their interests. For example, some colleges require a senior independent project: this could be a modeling project (perhaps with government or industry linkage) in an applied area of the student's interest, a research or scholarly investigation in an area of mathematics, or an independent study that includes writing about some area of mathematics. Other examples include problem-oriented senior seminars, an undergraduate colloquium series, or seminars in which students present

accessible journal articles. Most of these “capstone experiences” include reflection, writing, and oral communication of mathematics. These opportunities are sometimes the most rewarding and enlightening mathematical experience a major has, so they should be encouraged and supported in all appropriate circumstances.

Communication and Team Learning

Most mathematical sciences majors will eventually work in situations where success requires that they be able to explain mathematical work to non-mathematical audiences. In these situations the ability to communicate clearly and effectively about mathematics is vital. Undergraduate mathematical sciences programs should prepare students for communicating mathematics, both orally and in writing. Student presentations in mathematics clubs, in special seminars, and in regular classes can provide practice to help develop skills in communication. Similarly helpful are joint class projects requiring team work with results that are to be well-presented in standard English. In appropriate courses, students can be assigned to teams for collaborative investigation of mathematical problems of some complexity. Such problems should encourage verbal interaction among the team members as well as the written expression of mathematical ideas. Communication skills are further honed when upper-division mathematics students assist freshmen and sophomores in learning, or tutor high school students, perhaps in student service programs.

Independent Mathematical Learning

Whether a mathematics major enters a mathematical sciences career immediately after graduation, goes directly to graduate school, or enters another career path, the student will need to function as an independent learner. Imaginative curriculum and teaching can speed the development of this independent mathematical learning. Readily accessible methods include increasing the numbers of student presentations in junior and senior courses, or independent study projects in which students are expected to master a mathematical subject in the strictly elective part of the curriculum. (However, no part of the recommended component courses A through F should be replaced by an independent study activity.)

Structured Activities

On many campuses, a variety of organized activities are available to strengthen the undergraduate experience. Student success has been

shown to be supported by linking out-of-class activities with study [53]. Such supplementary activities must be set up in a way that is sensitive to the need to be inclusive of under-represented groups, and not just be neutrally available, if these students are to benefit.

For many years strong students have participated in mathematical honorary societies. On other campuses, mathematics club-type activities are available and include a broad range of students and interests. More recently, MAA has developed a Student Chapter program which encourages participation by all students with interests in the mathematical sciences. Within college honors programs, specialized mathematics activities may be available. Mathematical contests may provide valuable broadening with the opportunity for team work and independent learning. In addition to the long-established Putnam competition and the newer Mathematical Contest in Modeling [36], local and regional contests at both lower- and upper-division levels hone mathematical skills.

Undergraduate research experiences [43] including summer research programs are valuable both as learning experiences and as identification and recruiting devices to aid retention in the academic setting of talented students. Stipends are often available to students for summer research and internship activities; NSF funds a number of undergraduate research programs at universities, and government and industrial organizations have found that summer internships are advantageous both to the participating students and the sponsoring organizations. Internships during the academic year are less common in mathematics than in engineering, but where available they provide students both funds for future terms in school and valuable work experience. At some institutions, cooperative education experiences are available for interested students.

Additional Considerations

Our recommendations for the undergraduate major in the mathematical sciences arise out of current priorities and practice within the profession. The major depends on curriculum, on teaching practices, and on student experiences that move beyond strictly classroom learning. It must also be based on a realistic assessment of the needs of the profession—particularly for well-qualified and enthusiastic teachers of mathematics at all levels. The following sections outline some related concerns that the mathematical community must keep in mind in conjunction with the undergraduate major.

Secondary Teacher Preparation

Mathematical sciences departments have always played a significant role in the preparation of secondary teachers of mathematics. Courses designed according to the philosophies and practices described in this report will enrich the experiences needed by teachers who will be confronted with changing demands on the curriculum in the schools. The MAA's Committee on the Mathematical Education of Teachers [25] and the National Council of Teachers of Mathematics [31] both recommend that prospective secondary school teachers of mathematics complete a major in mathematics. Thus students who have selected secondary school teaching as a profession will select courses appropriate to this objective while satisfying the structural components of the mathematical science major. Meeting the needs of those students who seek teaching credentials can be facilitated by flexibility in the department's implementation of a component-based major. For example, courses in Euclidean and non-Euclidean geometry, number theory, or the history of mathematics are useful for prospective teachers and are often required for state certification; such courses can easily be included within an appropriate track structure.

Future Graduate Study

Any track in a mathematical sciences major following the component structure recommended in this report can lead to successful graduate study for students showing potential for such study. It is important that early specialization not preclude a strong foundation for future study. The concerns that a reasonable number of undergraduates pursue graduate work, that they form a demographic cross section, and that they be well-informed about ways to make the transition to graduate school smoothly, are discussed in other parts of this report and in [6, 21, 26, 44].

For students who may wish to pursue graduate study, the courses chosen to satisfy components **D** and **E** will be especially critical; an expectation of mathematical maturity and sophistication is justified for these students. Data from a survey [35] of the highest-ranked graduate departments (including some applied mathematics departments) show that—independent of the graduate specialty—72 of 76 reporting departments rated real analysis or advanced calculus as essential or highly recommended preparation for their programs.

Information Needs

To make wise choices, one needs good information about the effects of different options based on past experience. Unfortunately, in the area of undergraduate mathematics, good information is rare. Here are some examples of areas where much better information is urgently needed:

Teaching Methods. Information on effective and efficient teaching methods for the college mathematics classroom is sparse; while there is some information in journals and periodicals and some in the recent helpful MAA volumes (e.g., [7, 42, 56]), too little specific information of this kind is regularly collected and published. There are, however, many recommendations encouraging improvement and even commentary criticizing both teaching methods and the professoriate, not only in the popular papers but also in the professional literature. It is often recommended (e.g., [6, 25, 32, 33, 48]) that teaching methods other than lecturing should be used. To help those charged with curricula reform on campus, practical information is needed on effective teaching styles for various learner groups.

Educational Environment. Continued efforts should be encouraged for professional societies to collect and disseminate information about ways to improve the environment of the mathematical curriculum. Specific needs noted as this report was prepared include requests for information on the nurturing and encouragement of junior faculty, on evaluation of teaching effectiveness, and on advising strategies, particularly regarding job and further study opportunities for mathematics majors.

Major Requirements. There is not much current data about the actual status and requirements of mathematical sciences major programs in this country. The quintennial sample survey data of the Conference Board on the Mathematical Sciences (CBMS) provide partial snapshots, but not in sufficient detail about the major. It would also be helpful to have more information about subsequent experiences of former students.

Existing Tracks. More information needs to be gathered about existing tracks within the mathematical sciences major. Some tracks may impose only a few restrictions on the choice of courses within the structural components but require a number of courses outside the mathematical sciences. Other tracks may impose very specific requirements on the basic structural components but allow a high degree of choice otherwise. Dialogue between the mathematical community and the clients of various tracks would help ensure appropriate coordination of requirements. (For

example, discussion with actuaries may help align actuarial mathematics curriculum recommendations with the advice both of those who make the actuarial examinations and of those who hire actuarial trainees.)

Graduate School Preparation and Retention. National data and anecdotal information suggest that many U.S. undergraduates who enter graduate school do not persist to graduate degrees. If that is true, then it should be a high priority of professional societies, reflecting faculty concern, to recommend improvements. Curriculum planners at the undergraduate level and graduate faculty need better information if they are to work together effectively to improve success rates of beginning graduate students. (See [1, 13, 14, 15, 21, 26, 27].)

Statistics. A joint committee of the American Statistical Association (ASA) and the MAA is currently examining a wide range of issues, including the probability and statistics course suitable for mathematics majors and other undergraduate statistics courses taught in (joint) mathematical sciences departments. It would be helpful if this joint committee were to publish guidelines for an undergraduate statistics major or a statistics track within a mathematical science major.

Conclusions

Programs that follow the recommendations of this report will develop mathematical sciences graduates with broad experience and good prospects for using and continuing to learn mathematics. Whether a student intends to teach in a secondary school, work in business or industry, or pursue graduate study, an exciting and flexible program fashioned from the components of this major structure will be appropriate for long-term goals and unanticipated challenges. As with any set of rules intended to apply in diverse situations, flexibility may be warranted in special cases (such as majors within a mathematical sciences department that follow another professional society's guidelines).

The philosophy expressed in these recommendations embodies educational principles that will lead to an enriching educational experience, and the recommended program structure provides a flexible vehicle for fulfilling those principles. One underlying tenet, however, transcends the particular form of curriculum implementation: It is only by requiring substantive achievement of our students that we will be able to produce the sort of quantitatively expert individuals who are going to be the mainstay of the discipline and of society for the next century.

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