Mid-West Model Theory

In Honor of Charles Steinhorn's 70th Birthday

University of Illinois Chicago

October 14–16

Speaker: Sylvy Anscombe (Paris Cité)
Title: TBA
Abstract:

Speaker: Artem Chernikov (UCLA)
Title: Higher Vapnik–Chervonenkis theory
Abstract: Finite VC-dimension, a combinatorial property of families of sets, was discovered simultaneously by Vapnik and Chervonenkis in probabilistic learning theory, and by Shelah in model theory (where it is called NIP). It plays an important role in several areas including machine learning, combinatorics, mathematical logic, functional analysis and topological dynamics. We develop aspects of higher-order VC-theory, establishing a higher arity generalization of the epsilon-net theorem for sets (and functions) with bounded VC_k-dimension. As an application, we obtain a strong version of Szemerdi's regularity lemma for hypergraphs omitting a fixed finite k-partite k-hypergraph. Joint work with Henry Towsner.

Speaker: Gabriel Conant (Ohio State)
Title: Preservation of tameness when naming a structure
Abstract: A pervasive question in model theory is when desirable properties of some structure are preserved when expanding that structure by new definable sets. For example, over the last several years there has been an extensive study focused on preserving stability in expansions of the group of integers. An interesting phenomenon in this work is that all previously discovered examples of stable expansions of the group of integers are, in fact, superstable of U-rank omega. Thus it has remained an open problem to exhibit a strictly stable expansion of the group of integers. In this talk, I will present a solution to this problem, which is obtained in a much broader framework. Specifically, we will investigate preservation of model-theoretic tameness (e.g., stability, simplicity, NIP, NTP2, NSOP1) when expanding the induced structure on a definable stably embedded set by some arbitrary new structure. Joint work with C. d'Elbé, Y. Halevi, L. Jimenez, and S. Rideau-Kikuchi.

Speaker: Matthew DeVillbis (Ohio State)
Title: Generic differential equations are strongly minimal
Abstract: In this talk, I will outline a new technique for showing that nonlinear algebraic differential equations are strongly minimal. This is used to prove the strong minimality of generic differential equations with sufficiently large degree, answering a question of Poizat (1980). Time permitting, I will also discuss ongoing work in applying this method to differential equations of interest whose coefficients are not generic. This is joint work with James Freitag.

Speaker: Dario Garcia (Universidad de los Andes)
Title: Pseudofinite structures, simplicity and ordered asymptotic classes
Abstract: A structure M is said to be pseudofinite if every first-order sentence that is true in M has a finite model, or equivalently, if M is elementarily equivalent to an ultraproduct of finite structures. When restricted pseudofinite structures, Łos' Theorem provides a way to lift the counting measure on a class of finite structures to give notions of dimension and measure on their ultraproducts, thus allowing ideas from geometric model theory to be used in this context.

Quantitative properties in classes of finite structures also can induce desirable model-theoretic properties in their ultraproducts. For instance, Macpherson and Steinhorn defined in [MS] the concept of (1-dimensional) asymptotic classes, which are classes of finite structures that satisfy strong conditions on the sizes of definable sets. The most notable examples are the class of finite fields, the class of cyclic groups, or the class of Paley graphs. The infinite ultraproducts of asymptotic classes are all supersimple of finite SU-rank, but recent generalizations of this concept (known as *multidimensional asymptotic classes*, or m.a.c.) are more flexible and allow the presence of ultraproducts whose SU-rank is possibly infinite, or even structures that are NSOP but not simple (cf. [AMSW], [Wolf]).

In this talk we will present several results regarding model-theoretic properties of pseudofinite structures and classes of finite structures approximating them. In the first part I will give a brief overview of the subject, and present joint work with Macpherson and Steinhorn on which we explored conditions on the (fine) pseudofinite dimension (defined in [Hru]) on an ultraproduct M of finite structures that guarantee simplicity or supersimplicity of the theory of the underlying theory T=Th(M). In the second part of the talk I will present the notion of ordered asymptotic classes of finite structures which

is an adaptation of the concept of 1-dimensional asymptotic classes for the context of finite linearly ordered structures that combines ideas and results from asymptotic classes with concepts of o-minimality.

References:

[AMSW] S. Anscombe, D. Macpherson, C. Steinhorn, D. Wolf. Multi-dimensional asymptotic classes and generalised measurable structures. In preparation.

[Elwes] R. Elwes. Asymptotic classes of finite structures. Journal of Symbolic Logic, Volume 72, Issue 2, (2007)

[GMS] D. Garca, D. Macpherson, C. Steinhorn. Pseudofinite structures and simplicity. Journal of Mathematical Logic, 15, 1550002 (2015)

[Ga] D. Garca. Ordered asymptotic classes of finite structures. Annals of Pure and Applied Logic, 171, 4, (2020)

[Hru] E. Hrushovski. On pseudo-finite dimensions. Notre Dame Journal of Formal Logic, vol.53, No. 3-4, (2013)

[MS] D. Macpherson, C. Steinhorn. One-dimensional asymptotic classes of finite structures. Transactions of the American Mathematical Society, vol 360, No. 1 (2008)

[Wolf] D. Wolf. Multidimensional asymptotic classes of finite structures. PhD. Thesis. University of Leeds, (2016).

Speaker: Dugald Macpherson (Leeds)
Title: Definable sets in finite structures
Abstract: Chatzidakis, van den Dries and Macintyre proved in a 1992 paper a strong uniformity result on the approximate cardinalities of definable sets in finite fields. They generalise and use the Lang–Weil estimates on the number of rational points in a finite field of an absolutely irreducible variety defined over the field. The work yields an associated dimension-measure pair for definable sets in pseudofinite fields. Over the last 20+ years, Charles Steinhorn, I, and others have developed frameworks for which the CDM results provide a special and motivating case.

This talk is coordinated with that of Sylvy Anscombe. I will describe work with Anscombe, Steinhorn and Wolf on 'multidimensional asymptotic classes and 'multidimensional exact classes of finite structures. We consider classes of finite structures M such that for each formula $\phi(x, y)$, sets of the form $\phi(M, a)$ take a bounded number, independent of M, of approximate (or exact) cardinalities as the parameter tuple a ranges through M, with an

associated definability clause. Ultraproducts of such classes of finite structures are 'generalised measurable, and the talk of Anscombe will focus on the infinite (generalised) measurable context. I will describe examples and the basic theory developed so far for multidimensional asymptotic and exact classes of finite structures.

Speaker: Chris Miller (Ohio State)
Title: Expansions by cofinal sequences of expansions of dense linear orders
Abstract: The structure $(\mathbb{Q}, <, \mathbb{N})$ has many nice model-theoretic properties, in particular, it is "near model complete" and explicitly axiomatizable over $(\mathbb{Q}, <)$ in an obvious way. When can we similarly understand structures of the form $(\mathfrak{M}, P)$, where $\mathfrak{M}$ is an expansion of a dense linear order without endpoints $(M, <)$, and $(M, <, P) \vDash \mathfrak{Q}, <, \mathfrak{N})$? We are particularly interested in the model theory (not just the definability theory) of expansions of structures on the real additive group by (ranges of) cofinal sequences. Some of this is ongoing joint work with Alf Dolich and Trent Ohl.

Speaker: Alf Onshuus (Universidad de los Andes)
Title: Decomposable linearly ordered structures and classification of Lie groups definable in o-minimal structures
Abstract: In a paper from 2009, Steinhorn and I introduced the concept of "decomposable linearly ordered structures". This was the first of a planned series of papers (which never ended up happening), but which it was the first and crucial step for a series of results which concluded with a full understanding which real Lie groups can be "represented" in an o-minimal expansion of the real field.

In this talk I will give a survey of how this happened, highlighting the most important results, and explaining why the anticipated series of papers on decomposable ordered structures never ended up happening (or at least not yet).

Speaker: Adele Pagett (McMaster)
Title: Sublogarithmic-transexponential series
Abstract: I will introduce transseries and discuss a field of transseries built from a function that grows faster than any composition of exponential functions. Such a function is called transexponential. I will also discuss a connection to the open problem of whether there are transexponential o-minimal structures.

Speaker Kobi Peterzil (Haiffa)
Title: Additive, possibly unordered, reducts of a real closed field (joint work with Hind Abu Saleh)
Abstract: We returned to an old problem, first introduced to me by Charlie Steinhorn, about the possible reducts of real closed fields which still expand the underlying (unordered) vector space. Along the way we introduced the notion of a **strongly bounded structure**: a reduct of a linearly ordered structure in which every definable set in one variable is either bounded or co-bounded (e.g. rays are not definable).

We examine definable sets in strongly bounded reducts of o-minimal structures, and as a result give a complete list of the additive reducts of a real closed field.

Speaker: Anand Pillay (Notre Dame)
Title: Open subgroups of p-adic algebraic groups
Abstract: I will discuss a problem I asked around 1987; whether open subgroups of p-adic algebraic groups are (p-adic) semialgebraic. Joint work with Ningyuan Yao.

Speaker: Margaret Thomas (Purdue)
Title: Definable topological spaces in o-minimal structures
Abstract: A 'definable topological space' is a definable set equipped with a definable family which forms a basis for a topology on that set. In the spirit of earlier work of Onshuus, Ramakrishnan and Steinhorn on definable orders, that of the speaker on definable function spaces, and that of Walsberg on definable metric spaces, we present progress made towards understanding the nature of topological spaces definable in o-minimal structures. This is based on a long-term joint project with Pablo Andjar Guerrero and Erik Walsberg, and intersects with work carried out independently by Peterzil and Rosel. Our work includes the classification of certain definable topological spaces via decomposition and embedding theorems (including a definable version of an open conjecture from classical topology), and the identification of suitable definable analogues of classical notions, such as compactness, separability and metrizability, as well as universality results for certain classes of spaces.

Speaker: Mariana Vicaria (UCLA)
Title: Towards an Imaginary Ax-Kochen/Ershov principle
Abstract: One of the most striking results in the model theory of henselian valued fields is the Ax-Kochen theorem, which roughly states that the first

order theory of a finitely ramified henselian valued field is completely determined by the first order theory of the residue field and its value group. A model theoretic principle follows from this theorem: any model theoretic question about the valued field can be reduced into a question to its residue field, its value groups and their interaction in the field. Our leading question is: Can one obtain an Ax–Kochen style theorem to eliminate imaginaries in henselian valued fields?

Following the Ax–Kochen principle, it seems natural to look at the problem in two orthogonal directions: one can either make the residue field tame and understand the problems that the value group brings naturally to the picture, or one can assume the value group to be very tame and study the issues that the residue field would contribute to the problem. In this talk we will address the first approach. I will explain the sorts required to obtain elimination of imaginaries in henselian valued fields of equicharacteristic zero with residue field algebraically closed and more general value groups.