University of Illinois at Chicago

0. Motivation

Apply polyhedral homotopies to polynomial system arising in mechanisms design. Special structure: not so sparse but still mixed volume < degree bounds Five basic joints:

Revolute.	R	Prismatic.	Ρ	Cylindric [.]	C	Universal	Т	S	pherical.	S
	Ν	1 115matic.	T	Cymunc.	C	Universal.	T	N	phoneal.	D

Basic chains	Surface	Total deg	LPD bound	Mixvol
PRS	elliptic cylinder	2,097,152	247,968	125,888
RRS	circular torus	2,097,152	868,352	474,112
RRS	general torus	4,194,304	448,702	226,512



Figure 4.4: The elliptic cylinder reachable by a PRS serial chain.



Figure 4.8: The general torus reachable by the wrist center of an RRS serial chain.

1. Three Stages to Solve a Polynomial system $f(\mathbf{x}) = 0$

1. Compute mixed volume of the Newton polytopes spanned by the supports of f

- 2. Solve a random coefficient start system $g(\mathbf{x}) = 0$ which has the same monomials as with random coefficients and has exactly mixed volume isolated solutions.
- 3. Use $(1 t)g(\mathbf{x}) + tf(\mathbf{x}) = 0$ to solve $f(\mathbf{x}) = 0$.
- Stages 2 and 3 are computationally most intensive $(1 \ll 2 < 3)$.

References

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Parallel Implementation of the Polyhedral Homotopy Method

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2. Statio

Since poly same amo

Work La hedral hom int of work	oad Balancin notopies solve a g	generic sys	tem, we ex	pect every path to take the	3. Dyn For poly load bal	namic W ynomials v lancing is r	ork Lo which are needed.	ad Bala not so s	a no par
lgorithm: Δ_{ω} , put: Δ_{ω} , utput: G^{-}	Sketch of a paral $G(\mathbf{x}) = 0$.mixed $-1(0)$.	llel version - <i>cell con£g</i>	of polyhe guration and all solut	dral homotopies. In d generic system ions to $G(\mathbf{x}) = 0$	Algorith Input	hm: Dynar t: \triangle_{ω}, V, p ut: $G^{-1}(0)$	nic distril	oution of	ce m
Manag read inpu broadcast distribute collect solu write to	gerWorkat £ledata \rightarrow receivecells \rightarrow receivetrack putions \leftarrow send sol£le	xers e data constant cells state oaths utions	lata = syst atic worklo compute	em and lifting oad distribution e solutions	if #△ distr else distr distr distr	$\Delta_{\omega} \leq p$ then the the formula of V pathesis of V pathe	n hs; orst $\# \triangle_{\omega}$ ast cell;	- 1 cells	>,
ice the stati	ic distribution of	the cells w	vith an exa	mple.	Algorit	hm: Dynan $\cdot C(\mathbf{x}) = 0$	nic distril	oution of	ce.
manager	r worker	1 w	orker 2	worker 3		• $G(\mathbf{X}) = \mathbf{C}$	J , V .		
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with 13 wo	orkers, with static	load distri	ibution.			1	50.7021	-	
	Problem	#Paths	CPU Ti	me		2	24.51/2	$\begin{array}{c c} 2.1 \\ 2.9 \end{array}$	
	cyclic 6-roots	156	0.1	19m			10.303U	2.0 3 A	
	cyclic 7-roots	924	0	30m		5	11.6913	4.3	
	cyclic 8-roots	2,560	0.7	78m		6	10.3779	4.9	

We introd

c Work L yhedral hom ount of work	oad Balancin otopies solve a g	g generic sys	tem, we expec	ct every path to take the	3. Dynamic W For polynomials v load balancing is 1	ork Los which are needed.	ad Bala not so s	ano spai
Algorithm: S Input: \triangle_{ω} , (Output: G^- Manage read input broadcast distribute collect solut write to a luce the stati	Sketch of a paral $G(\mathbf{x}) = 0.mixed$ $^{1}(0).$ er Work t file data \rightarrow receive cells \rightarrow receive track p stions \leftarrow send sol file c distribution of	Ilel version -cell con£g -cell con£g $cers$ $cers$ $cells$ sta $cells$ sta $oaths$ $utions$ $the cells$	of polyhedra guration and g all solution data = system atic workload compute so	I homotopies. generic system as to $G(\mathbf{x}) = 0$ and lifting distribution blutions	Algorithm: Dynar Input: \triangle_{ω}, V, p Output: $G^{-1}(0)$ if $\# \triangle_{\omega} \leq p$ then distribute V particles else distribute the factorial end if. Algorithm: Dynar	nic distribution $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ 	oution of	ce <i>m</i> 5; ce
manager	worker	1 w	orker 2	worker 3	$\frac{\text{Input: } G(\mathbf{x}) = 0}{\mathbf{d}0}$	J , <i>V</i> .		
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total #paths :	: 41 track paths	: 14 track	t paths : 14 tr	rack paths : 13	#workers	Static	Speedur	
e for start sys	stems to solve the	he cyclic n	-roots probler	ns, using a cluster con-		50.7021		
i witii 15 woi	Droblom	$\frac{\text{#Detha}}{\text{HDetha}}$			2	24.5172	2.1	
_	Problem avalia 6 reata		CPU IIme		3	18.3850	2.8]
-	cyclic o-roots	$\frac{130}{024}$	0.191	$\frac{\Pi}{n}$	4	14.6994	3.4]
-	cyclic & roots	724 2560	0.301	$\frac{11}{n}$	5	11.6913	4.3]
	cycne 0-10018	<i>4,</i> ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	U. / OL	11	6	10.2770	10	

Wall time £guration

Problem	#Paths	CPU Time
cyclic 6-roots	156	0.19m
cyclic 7-roots	924	0.30m
cyclic 8-roots	2,560	0.78m
cyclic 9-roots	11,016	3.64m
cyclic 10-roots	35,940	21.33m
cyclic 11-roots	184,756	2h 39m
cyclic 12-roots	500,352	24h 36m



More References

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Wall time for mechanism design problems on our cluster and argo.

5.2

6.5

6.8

7.3

7.7

7.9

9.6877

7.8157

7.5133

6.9154

6.5668

6.4407

5.1462 9.8

8

10

12

10 13

	Boun	ds on #Soluti	ons	dynamic load distribution		
Surface	Total deg	LPD bound	Mixvol	our cluster	time on argo	
elliptic cylinder	2,097,152	247,968	125,888	11h 33m	6h 12m	
circular torus	2,097,152	868,352	474,112	7h 17m	4h 3m	
general torus	4,194,304	448,702	226,512	14h 15m	6h 36m	

4. Conclusion

A static work load distribution provides already a decent speedup of the polyhedral homotopies on a cluster computer for "small" or very sparse system. However, for polynomial systems which are not so sparse as the mechanisms design problems, dynamic load balancing is needed.

cing

rse as mechanisms design problems, dynamic

ells executed by the manager.

nixed-cell configuration, volume, #processors all solutions to $G(\mathbf{x}) = \mathbf{0}$ *distribute path by path*

> distribute cell by cell *distribute path by path*

ells executed by all the workers. **Output:** a subset of $G^{-1}(\mathbf{0})$.

receive cell

s) = 0; perform coordinate transformation one linear system to solve track Vol(C) paths or just one path message to manager reporting results

roblem for an increasing number of workers

		0	
nic on o	ur cluster	Dynamic	on argo
ynamic	Speedup	Dynamic	Speedup
53.0707		29.2389	
25.3852	2.1	15.5455	1.9
7.6367	3.0	10.8063	2.7
2.4157	4.2	7.9660	3.7
0.3054	5.1	6.2054	4.7
9.3411	5.7	5.0996	5.7
8.4180	6.3	4.2603	6.9
7.4337	7.1	3.8528	7.6
6.8029	7.8	3.6010	8.1
5.7883	9.2	3.2075	9.1
5.3014	10.0	2.8427	10.3
4.8232	11.0	2.5873	11.3
4.6894	11.3	2.3224	12.6