Circle Centers, the Common Core, and authentic problems

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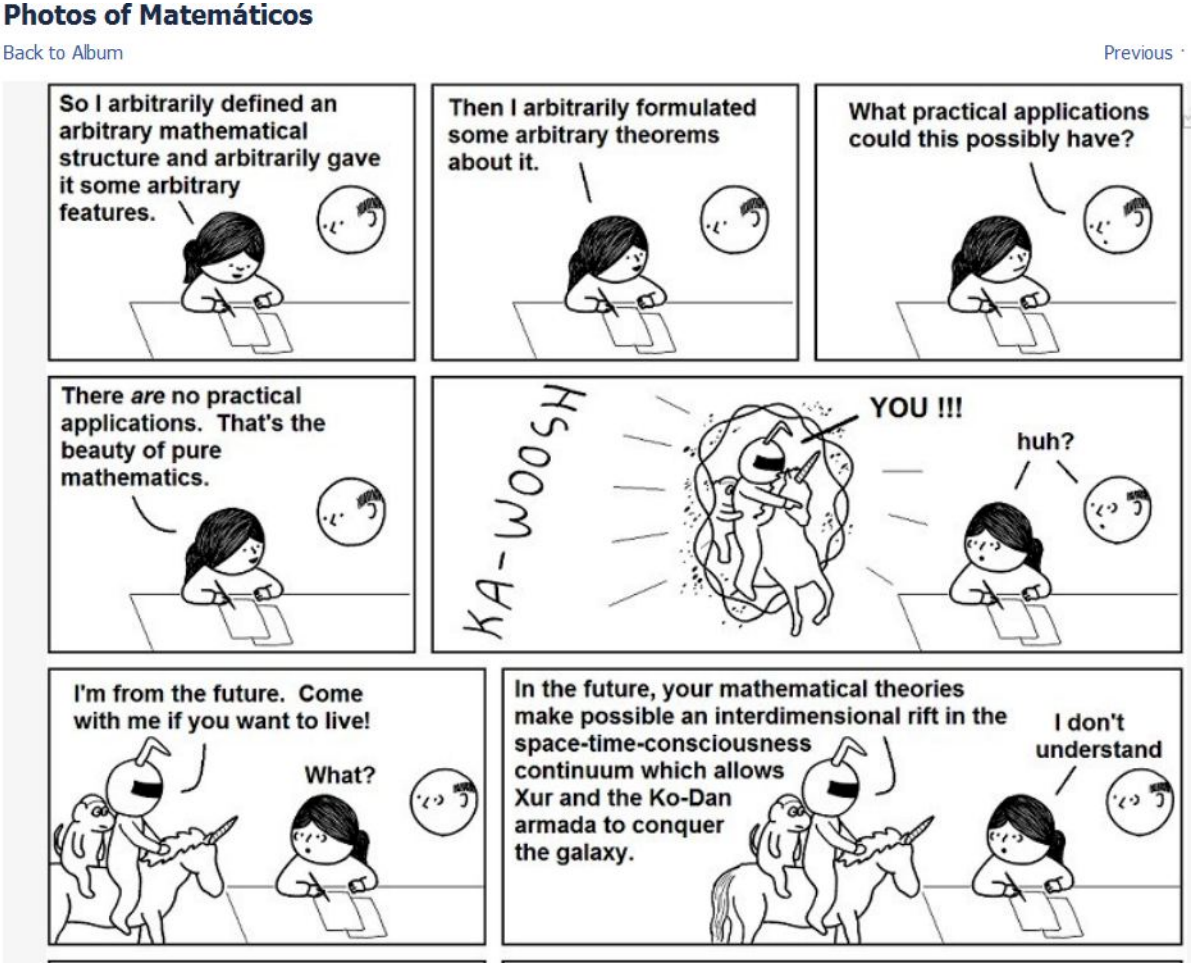
Outline

Mathematics: Pure and Applied



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Agenda

1. Concurrencies and CCSS
2. Circumcenter
3. Authentic ?? Problems
4. Transformations and Concurrencies
5. Incenter
6. pure and applied motivations

Math Content Standard

G-C.3 3

Understand and apply theorems about circles

G-C.3 3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

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Math Practice 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose,

Math Practice Standard

Math Practice 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. [. . .]

By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. [. . .]

They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Make sure students connect circles with the world

Measure the circumference of a circular wastepaper basket (in inches or centimeters). Explain how you made the measurement and what your answer is.
Measure the diameter of the same wastebasket. Explain how you made the measurement and what your answer is.

What is a circle?

How many points determine a circle.

Andreas: Wembo-Nyama Crater problem

Discussion of Wilson note

`http://jwilson.coe.uga.edu/CntrTriLrngTsk/CenTriLrngTsks.html`

Some questions to think about

What is a concurrency and how many concurrencies are there?

Why do we study them?
What is the role of applications in studying circles?

When is the distinction between 'segment' and 'line' important?

How do we develop students ability to understand the difference between precision and pedantry?

Or is that too much to ask?

What is an authentic problem

Do you believe the following definition?

Educause learning initiative

What Is Authentic Learning? Authentic learning typically focuses on real-world, complex problems and their solutions, using role-playing exercises, problem-based activities, case studies, and participation in virtual communities of practice. The learning environments are inherently multidisciplinary. They are not constructed in order to teach geometry or to teach philosophy.
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I dispute this. An authentic problem is one which presents a real world situation in an attempt to illustrate a mathematical concept. The problem/solution pair is **not** authentic if the intended solution does not reflect the actual practical problem.

Are these problems authentic

The airport problem

http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=13&ved=0CHYQFjAM&url=http%3A%2F%2Fmath.kennesaw.edu%2Fmathed%2FNMMC%2FPoints%2520of%2520Concurrency%2520in%2520Triangles%252010-08-06%2520Student.doc&ei=aiNnUY_FJqHi2QXlw4DIAQ&usg=AFQjCNHvZVzVWuQxk1FatE3OGsbxzjuflw&sig2=168MrkbM70O2PHdf21OHiQ&bvm=bv.45107431,d.b2I--

Read and work on the problems here. The main question is not.

How do you do this problem?

But, is it authentic?

Cell tower problem

Recall that the problem of finding the circumcenter as the most economical place for three people in different towns to meet for dinner is silly.

Is this problem authentic?

The cell towers at points A, B, C all send a signal to George's cell phone is exactly the same time. Find where George lives.

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Is this problem any better? Why or why not?

Fermat Point

Find the point so as to minimize the sum of the distances to the three vertices of a triangle.

Directions

1. Draw a triangle $\triangle ABC$ and then draw an equilateral triangle on each side of ABC.
2. Draw a line from each vertex of the original triangle to the vertex of the opposite equilateral triangle.
3. Conjecture and prove.

Understanding Centers Via Transformations

Andreas

Incenter

Do the first 3 problems on the incenter handout.
How can you find a point that minimizes the sum of the distances to the sides of a triangle.

How to solve the problem

Draw a triangle on geogebra with the measurements of the distances to sides and try to minimize.

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Draw a triangle on geogebra with the measurements of the distances to sides and try to minimize.
Wilson says the point that minimizes the sum of the distances to the sides of a triangle is the vertex of the shortest altitude.

Sketch and argue why this is true.

How not to solve the problem

http://www.physicsforums.com/archive/index.php/t-529838.html

A mathematical motivation for incenters

Before considering problem 4 on the incenter activity, we will review some ideas from the fall workshop.

From geometry to numbers

We want to define the addition and multiplication of segments.

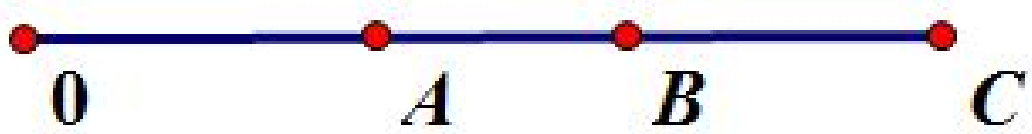
1. identify the collection of all congruent line segments as having a common 'length'. Choose a representative segment OA for this class .
2. define the operation on such representatives.

Today we do step 2; the variant of step 3 will be an extension.

Defining segment addition

Adding line segments

The sum of the line segments AB and AC is the segment AD obtained by extending AB to a straight line and then choose D on AB extended (on the other side of B from A) so that $BD \cong AC$.

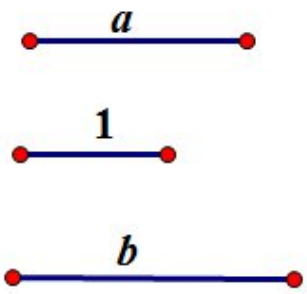


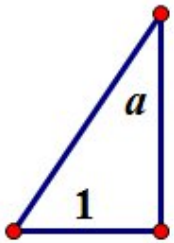
Defining Multiplication

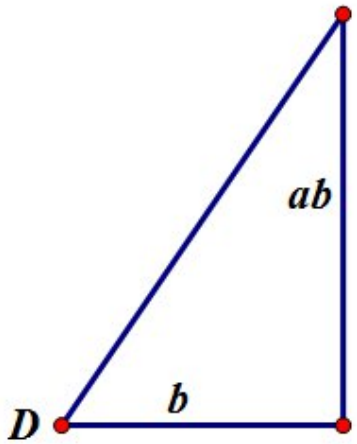
Consider two segment classes a and b. To define their product, define a right triangle[1] with legs of length a and b. Denote the angle between the hypoteneuse and the side of length a by α. Now construct another right triangle with base of length b with the angle between the hypoteneuse and the side of length b congruent to α. The length of the vertical leg of the triangle is ab.

[1] The right triangle is just for simplicity; we really just need to make the two triangles similar.

Defining segment Multiplication diagram







Sides of similar triangles are proportional

Figure out the proof sketched in problem 5 of the incenter handout.

Authenticity Again

Challenge

Find an authentic problem that illustrates incenter. That is we want a good reason to find a point that is equidistant from the three sides of a triangle.

Summary discussion

What, when, and why should we teach about circles?