A level set method for the semiclassical limit of the Schrödinger equation with discontinuous potentials

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Abstract

We propose a level set method for the semiclassical limit of the Schrödinger equation with discontinuous potentials. The discontinuities in the potential corresponds to potential barriers, at which incoming waves can be partially transmitted and reflected. Previously such a problem was handled by Jin and Wen using the Liouville equation– which arises as the semiclassical limit of the Schrödinger equation– with an interface condition to account for partial transmissions and reflections. However, the initial data are Dirac-delta functions which are difficult to approximate numerically with a high accuracy. In this talk, we extend the level set method introduced in (S. Jin, H. Liu, S. Osher and R. Tsai) for this problem. Instead of directly discretizing the Delta functions, our proposed method decomposes the initial data into finite sums of smooth functions that remain smooth in finite time along the phase flow, and hence can be solved much more easily using conventional high order discretization schemes.