Whitham averaging and nonlinear stability
of periodic traveling waves of viscous balance laws

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Abstract: For gas or elastic flows where thermodynamic stability is violated- for a van der Waal material for example, or for a viscous relaxation system for which the subcharacteristic condition fails- there may appear solitary-wave and periodic traveling wave solutions along with the more familiar traveling front, or shock wave, solutions. Behavior of a perturbed periodic wave train is described formally by the Whitham modulation equations. It has been shown by Serre and Oh-Zumbrun using Evans function techniques that this formal approximation may be connected rigorously to spectral stability. However, the connection between spectral and nonlinear stability until recently remained unclear.

In this talk, we show how to resolve this issue by finding a deeper connection to the Whitham system at the level of eigenmodes rather than just eigenvalues. With the resulting sharpened estimates we are able to show that spectral stability implies nonlinear bounded stability from $L1 \cap H^s$ to L^∞, and asymptotic convergence in H^s to an appropriately modulated wave. The resulting stability theory is strikingly parallel to that of traveling fronts, even though the associated behavior is quite different in appearance. We conclude by describing applications to periodic solutions both stable and unstable of the viscous Saint Venant equations for shallow water flow down an incline, a viscous balance law with relaxation.