

Homework #7

Due on 10/21/2026, Wednesday, before class.

1. Exercise 7.3.3 on page 430

7.3.3. If X_1, X_2 is a random sample of size 2 from a distribution having pdf $f(x; \theta) = (1/\theta)e^{-x/\theta}$, $0 < x < \infty$, $0 < \theta < \infty$, zero elsewhere, find the joint pdf of the sufficient statistic $Y_1 = X_1 + X_2$ for θ and $Y_2 = X_2$. Show that Y_2 is an unbiased estimator of θ with variance θ^2 . Find $E(Y_2|y_1) = \varphi(y_1)$ and the variance of $\varphi(Y_1)$.

2. Exercise 7.3.6 on page 430

7.3.6. Let $X_1, X_2, \dots, X_n$ be a random sample from a Poisson distribution with mean θ . Find the conditional expectation $E(X_1 + 2X_2 + 3X_3 \mid \sum_1^n X_i)$.

3. Exercise 7.4.1 on page 433

7.4.1. If $az^2 + bz + c = 0$ for more than two values of z , then $a = b = c = 0$. Use this result to show that the family $\{b(2, \theta) : 0 < \theta < 1\}$ is complete.

4. Exercise 7.4.2 on page 433

7.4.2. Show that each of the following families is not complete by finding at least one nonzero function $u(x)$ such that $E[u(X)] = 0$, for all $\theta > 0$.

(a)

$$f(x; \theta) = \begin{cases} \frac{1}{2\theta} & -\theta < x < \theta, \\ 0 & \text{elsewhere.} \end{cases} \quad \text{where } 0 < \theta < \infty$$

(b) $N(0, \theta)$, where $0 < \theta < \infty$.

5. Exercise 7.4.5 on page 434

7.4.5. Show that the first order statistic Y_1 of a random sample of size n from the distribution having pdf $f(x; \theta) = e^{-(x-\theta)}$, $\theta < x < \infty$, $-\infty < \theta < \infty$, zero elsewhere, is a complete sufficient statistic for θ . Find the unique function of this statistic which is the MVUE of θ .