Quiz3 - Math 313 **- Fall** 2014

1. (a) Suppose that $a_n \geq 0$ forms an increasing, bounded sequence for n in the natural numbers. The Completeness Axiom for the Real Numbers implies that such a sequence must have a limit. Give an example of such a sequence. Give a second example of a **bounded** sequence of positive real numbers that *does not* have a limit.

   (b) Find a rational number P/Q such that it is correct to say that

$$P/Q = .11111\cdots,$$

   and prove that you are right.

   (c) (Extra Credit) Give an injective map

$$F : P(\mathcal{N}) \longrightarrow \mathcal{R}$$

   where $\mathcal{N}$ denotes the natural numbers, $P(\mathcal{N})$ denotes the set of subsets of the natural numbers, and $\mathcal{R}$ denotes the real numbers. **Hint:** Every subset of the natural numbers can be uniquely represented by an infinite sequence of 0's and 1's.