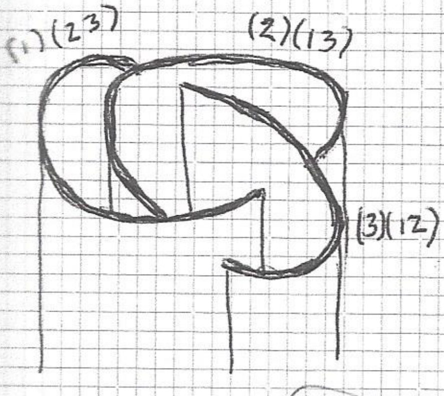
of the knot 3_1 (illustrated by Brauner in 1928). ONE (of 3

It consists of 3 3-cells, cut open along a self-intersecting wall under the knot, which are sewn together along the 3 2-cells under the knot. This results in 9 2-cells in the covering space. The 3 vertical 1-cells (where the wall intersects itself) lift to 9 1-cells in the cover, but the 3 horozontal 1-cells (running along the knot) lift to just 6 in the cover, due to branching.



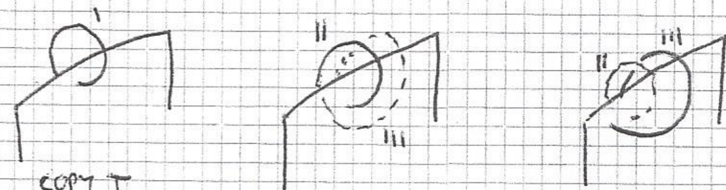
CONSIDER, FOR EXAMPLE, THE 3 2-CELLS UNDER THE SEGMENT THAT GETS SEWN TOGETHER BY THE PERMUTATION (1)(23) – I.E. COPIES II AND III ARE SEWN TO EACH OTHER BUT COPY I IS SEWN TO ITSELF. HERE'S WHAT IT LOOKS LIKE IN THE COVERING SPACE AT THE UPPERMOST CROSSING OF THE KNOT DIAGRAM:

IN COPY I — IN COPY II — IN COPY III

passing through it, one stays in copy I

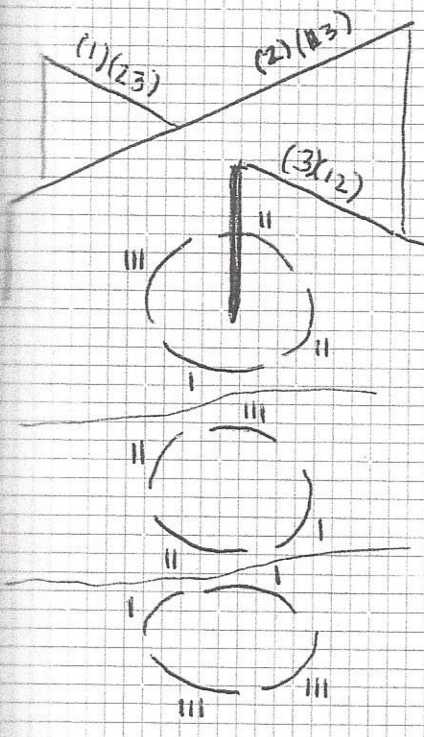
passing through these, in either direction, puts one in a different copy

SO A MERIDIAN AROUND THIS SEGMENT IN THE BASE SPACE LIFTS TO JUST TWO MERIDIANS IN THE COVER, ONE AROUND EACH OF THE BRANCH CURVES



COPY I — COPY II — COPY III

UNDER THE VERTICAL 1-CELLS ONE GETS (FOR DIFFERENT PERMUTATIONS AT A CROSSING) THE FOLLOWING PICTURE



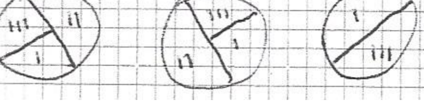
THERE ARE ONLY 6 1-CELLS RUNNING ALONG THE LINK ABOVE THE KNOT (IN THE COVER)

AND THE THREE OF THEM THAT ARE THE BRANCH OF INDEX 2 GIVE, ~~[illegible]~~ ALONG EACH SEGMENT, A HOMOLOGY RELATION BETWEEN THE TWO 2-CELLS UNDER EACH

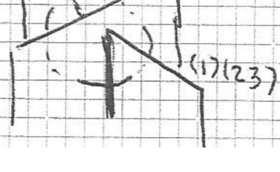
(THEY ARE THE INVERSE OF EACH OTHER)

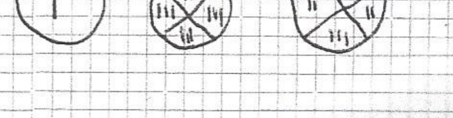
AND THREE RELATIONS AMONG THE 9 2-CELLS THAT MEET THEM.

A LITTLE DISC PERPENDICULAR TO EACH CELL PASSES THROUGH COPIES OF 3-SPACE IN A CLEARLY DEFINED WAY



FOR CROSSINGS WITH JUST ONE PERMUTATION THE PICTURE IS A BIT DIFFERENT





THESE LITTLE DISCS (OR CIRCLES AROUND THE 1-CELLS IN THE COVERING) CAN BE EASILY IMAGINED FROM A KNOT DIAGRAM WITH PERMUTATIONS WRITTEN ALONGSIDE THE SEGMENTS. WHEN WORKING WITH TRANSPOSITIONS ONLY, IT DOESN'T MATTER MUCH WHAT SIDE OF A SEGMENT YOU WRITE THEM ON. IN GENERAL, HOWEVER, THE DIAGRAM SHOULD BE GIVEN AN ORIENTATION AND PERMUTATIONS WRITTEN CONSISTENTLY ON ONE SIDE OR THE OTHER. (THIS IS HOW RILEY DID IT IN HIS 1971 PAPER "HOMORPHISMS OF KNOT GROUPS ON FINITE GROUPS", ALTHOUGH HIS ILLUSTRATION FOR THE 5-FOLD ALTERNATING COVER OF CONWAY'S KNOT GOT SCREWED UP.)

WITH A BIT OF PRACTICE, ALL THE COPIES OF S^3 THAT ARE SEWN TOGETHER TO CONSTRUCT A NON-CYCLIC COVER CAN BE "SEEN", SUPERIMPOSED UPON EACH OTHER, JUST FROM A DIAGRAM WITH PERMUTATIONS WRITTEN NEXT TO ORIENTED SEGMENTS.

THIS MODEL IS QUITE WELL-BEHAVED. ASSIGNED PERMUTATIONS PASS UNDER EACH OVERPASSING SEGMENT BY CHANGING EACH ELEMENT IN ACCORDANCE WITH THE PERMUTATION ASSIGNED TO THE OVERPASS. 2-CELLS IN THE COVER ARE BIUNIQUELY AND CONSISTENTLY IDENTIFYABLE WITH EACH ELEMENT OF THE PERMUTATION ASSIGNED TO THE SEGMENT ABOVE THEM, AND THEY ARE CONNECTED TO EACH OTHER IN THE OBVIOUS WAYS.

FROM THIS IT'S QUITE EASY TO ~~DERIVE~~ SOLVE A SET OF LINEAR EQUATIONS RELATING COEFFICIENTS ASSIGNED TO THE 2-CELLS THAT MAY OR MAY NOT ~~BO~~ BOUND ONE OF THE BRANCH CURVES (OR A MULTIPLE OF IT), DEPENDING ON WHETHER THE EQUATIONS ARE CONSISTENT OR HAVE A SOLUTION IN INTEGERS* THIS IS, HOWEVER, A LOT OF BORING (THOUGH SIMPLE) ALGEBRA, BEST LEFT TO A COMPUTER.

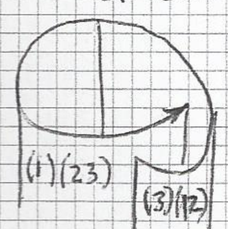
*OR RATIONAL NUMBERS

ALTERNATIVELY, ONE CAN JUST LOOK AT THE MODEL AND TRY TO FIND A SURFACE THAT COBOUNDS A BRANCH CURVE OF MULTIPLE BRANCHING INDEX. HERE THERE IS NO NEED TO CONFINE ONE'S SEARCH TO THE 2-CELLS OF THE DECOMPOSITION.

IF, FOR EXAMPLE, ONE LOOKS AT HEEGAARD'S ~~EXAMPLE~~ COVER OF THE TREFOIL, IT'S CLEAR THAT THE INDEX 2 BRANCH CAN BE APPROACHED ALONG ANY SEGMENT FROM 2 OF THE 3 COPIES OF S^3.

OVER A SEGMENT LABELED (1)(23), THE INDEX 2 BRANCH RUNS THROUGH COPIES II AND III, SO A SURFACE IN EITHER WILL CO-BOUND IT.

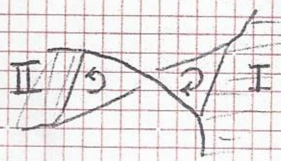
A CLOSER LOOK AT WHAT'S GOING ON IN A GIVEN COPY OF 3-SPACE IS THE FOLLOWING

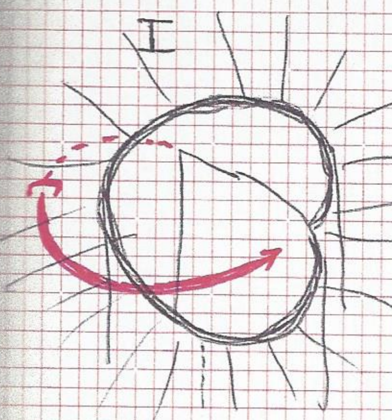


COPY II

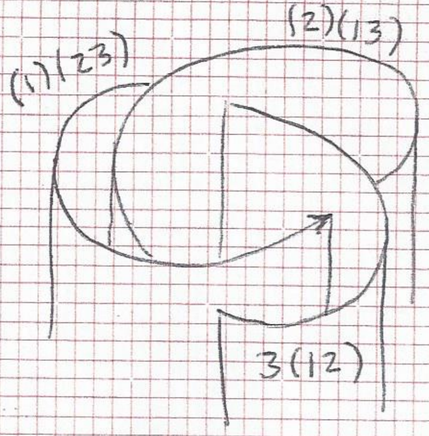
ONLY THE WALLS UNDER 2 OF THE SEGMENTS GET SEWN TO DIFFERENT COPIES, SO THE THIRD WALL POSES NO OBSTRUCTION TO INSERTING A DISC THAT COBOUNDS HALF OF THE BRANCH CURVE. JUST POP IT IN AND NOTE THAT THE PORTION OF IT'S BOUNDARY THAT HITS A WALL CAN BE PASSED ALONG A LITTLE TWISTED RECTANGLE TO A DIFFERENT COPY, WHEREIN THE REST OF THAT BRANCH CURVE MAY BE SIMILARLY COBOUNDED.

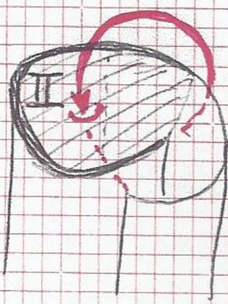
LITTLE TWISTED RECTANGLE





COPY I





COPY II

INDEX 1 BRANCH — IT OBVIOUSLY INTERSECTS THE COBOUNDING SURFACE −1 TIMES IN EACH OF COPIES I AND II

SO THE LINKING NUMBER BETWEEN BRANCH CURVES IS −2

SINCE THIS (OR SOMETHING EXACTLY LIKE IT) IS THE ONLY 3-FOLD IRREGULAR COVERING OF THE TREFOIL, THE KNOT IS NON-AMPHICHEIRAL.

NOTE THAT THE SIGN OF THE LINKING NUMBER IS THE SAME AS THE SIGN OF THE WRITHE.

P.S. IT'S EASIER TO SEE THIS FROM THE KNOT'S OTHER PROJECTION

IN MY SENIOR THESIS I COMPUTER-CALCULATED SEVERAL HUNDRED LINKING NUMBERS IN 3-FOLD DIHEDRAL COVERING SPACES. ABOUT HALF OF THEM CAN BE EASILY SEEN IN THIS WAY