CS 501 / MCS 501 – Computer Algorithms I

Fall 2020

Problem Set 1

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Due: 9/16/20 by the beginning of class

Instructions: Atop your problem set, write your name and also write the names of all the students with whom you have collaborated on this problem set.

1. [10 pts] Read jeremykun.com/2014/08/26/when-greedy-algorithms-are-perfect-the-matroid/. Then answer question 2.12 in the textbook on matroids.

2. [10 pts] Give as fast an algorithm as you can for finding *all* min-cuts in a graph that succeeds with probability $1 - o(1)$. (Note that deterministic algorithms also satisfy this requirement, so consider those.) Analyze its running time and argue for its correctness.

3. [10 pts] For the metric k-center problem, what is the largest possible ratio between the optimal value when we either require all cluster centers to be data points versus when we allow arbitrary points (sometimes called "Steiner points"), for which the distances are somehow specified, to be centers? Prove your answer correct.

4. [10 pts] While most clustering problems are NP-Hard, it is possible to formulate clustering objectives which traditional algorithms can solve exactly in polynomial time. Given objects $p_1, \ldots p_n$, and distances $d(\cdot, \cdot)$ on them (with $d(p_i, p_i) = 0$, $d(p_i, p_j) = d(p_j, p_i)$, and $d(p_i, p_j) > 0$ for $i \neq j$), consider the clustering problem of dividing the objects into k sets so as to maximize the minimum distance between any pair of objects in distinct clusters. Give a polynomial time algorithm for solving this version of the clustering problem. (Note that k is part of the input and can also grow with n.)