

Math 215, Fall 2026, Problem Set 1

- 0) [50 pts] Discuss some topics relevant to this homework (e.g. truth tables) with an LLM of your choice. Ask some interesting questions, seek clarification, and/or probe for more depth. Make the conversation interactive and show that you have given some thought to the LLM's responses. Turn in that chat log as your answer to this question. If you use an LLM for any of the remaining questions on this problem set, also turn in the corresponding chat logs.

- 1) [10 pts] Consider the following statement:

All books on this shelf are interesting.

Which of the following statements is the negation of the statement above?

1. No books on this shelf are interesting.
2. Every book on this shelf is not interesting.
3. There is a book on this shelf that is not interesting.
4. Every interesting book is on this shelf.
5. There are no uninteresting books on this shelf.

Can you find any statements in this list which have the same logical meaning as the original statement?

- 2) [10 pts] Use truth tables to prove that for all statements P and Q , the following statements are true:

1. $P \Rightarrow (P \text{ or } Q)$.
2. $(P \text{ and } Q) \Rightarrow P$.

- 3) [10 pts] Prove that 9 does not divide 106 using the definition of divisibility. Do not simply calculate $106/9$.

- 4) [10 pts] Consider the following statement, where n is an integer:

$6 \text{ divides } n \Rightarrow 3 \text{ divides } n$.

- (a) State this implication in words using “if ... then.”
 - (b) State it using “only if.”
 - (c) State it using the word “necessary.”
 - (d) Write its converse. Is the converse true? Explain.
- 5) [10 pts] Determine whether each statement is true or false for all integers n . If it is false, give a counterexample.
- (a) $n > 3 \Rightarrow n^2 > 9$.
 - (b) $n^2 > 9 \Rightarrow n > 3$.
 - (c) $n \text{ is even} \Rightarrow n^2 \text{ is even}$.