

Math 215, Fall 2026, Problem Set 3

- 0) [50 pts] Discuss some topics relevant to this homework (e.g. sets, quantifiers) with an LLM of your choice. Ask some interesting questions, seek clarification, and/or probe for more depth. Make the conversation interactive and show that you have given some thought to the LLM's responses. Turn in that chat log as your answer to this question. If you use an LLM for any of the remaining questions on this problem set, also turn in the corresponding chat logs.

- 1) [10 pts] Let

$$A = \{n \in \mathbb{Z} : -2 < n \leq 3\}$$

and

$$B = \{n \in \mathbb{Z} : n^2 < 10\}.$$

List the elements of A and B , and then find $A \cap B$, $A \cup B$, $A - B$, and $B - A$.

- 2) [10 pts] Let $A_1, \dots, A_n$ be subsets of a universal set U . Using induction on n , prove that for all integers $n \geq 2$,

$$(A_1 \cup A_2 \cup \dots \cup A_n)^c = A_1^c \cap A_2^c \cap \dots \cap A_n^c.$$

You may use the two-set De Morgan law

$$(A \cup B)^c = A^c \cap B^c.$$

- 3) [10 pts] Let A and B be subsets of a universal set U . Prove that

$$A \subseteq B \iff B^c \subseteq A^c.$$

For each direction, prove the implication using its contrapositive.

- 4) [10 pts] For each of the following statements:

- (a) write its negation using quantifiers, with the word “not” applied only to the predicate rather than to an entire quantified statement;
- (b) determine whether the original statement is true or false.

For any statement you claim is false, explain why.

$$(i) \quad \forall n \in \mathbb{Z}, \quad n^2 \geq 0.$$

$$(ii) \quad \exists n \in \mathbb{Z}, \quad n^2 = 2.$$

$$(iii) \quad \forall n \in \mathbb{Z}, \quad \exists m \in \mathbb{Z}, \quad m > n.$$

- 5) [10 pts] Prove or disprove each of the following statements:

$$(a) \quad \forall n \in \mathbb{Z}, \quad \exists m \in \mathbb{Z} \text{ such that } n < m^2,$$

$$(b) \quad \exists m \in \mathbb{Z}, \quad \forall n \in \mathbb{Z} \text{ such that } n < m^2.$$

Explain in words what each statement says and why changing the order of the quantifiers changes the meaning.