

Math 215, Fall 2026, Problem Set 4

- 0) [50 pts] Discuss some topics relevant to this homework (e.g. functions, composition of functions, sequences, injectivity, surjectivity, or inverse functions) with an LLM of your choice. Ask some interesting questions, seek clarification, and/or probe for more depth. Make the conversation interactive and show that you have given some thought to the LLM's responses. Turn in that chat log as your answer to this question. If you use an LLM for any of the remaining questions on this problem set, also turn in the corresponding chat logs.

- 1) [10 pts] Define functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = x^2 + 1 \quad \text{and} \quad g(x) = x - 1.$$

Find formulas for $f \circ g$ and $g \circ f$. Then find all $x \in \mathbb{R}$ such that

$$(f \circ g)(x) = (g \circ f)(x).$$

- 2) [10 pts] A sequence (a_n) of real numbers is defined by

$$a_n = \frac{2}{n+1}.$$

Using the definition of a null sequence, prove that (a_n) is null.

- 3) [10 pts] Let

$$\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} : x \geq 0\}.$$

Consider the following three functions, all given by the same formula:

$$\begin{aligned} f : \mathbb{R} &\rightarrow \mathbb{R}, & f(x) &= x^2, \\ g : \mathbb{R} &\rightarrow \mathbb{R}_{\geq 0}, & g(x) &= x^2, \\ h : \mathbb{R}_{\geq 0} &\rightarrow \mathbb{R}_{\geq 0}, & h(x) &= x^2. \end{aligned}$$

For each function, determine whether it is injective, surjective, both, or neither. Justify each answer with a proof or a counterexample.

- 4) [10 pts] Suppose that

$$f : X \rightarrow Y \quad \text{and} \quad g : Y \rightarrow Z$$

are functions.

- (a) Prove that if f and g are injective, then $g \circ f$ is injective.
- (b) Prove that if f and g are surjective, then $g \circ f$ is surjective.

- 5) [10 pts] Define

$$f : \mathbb{R} \setminus \{2\} \longrightarrow \mathbb{R} \setminus \{1\}$$

by

$$f(x) = \frac{x+1}{x-2}.$$

Find a formula for f^{-1} . Then prove that your proposed inverse really is the inverse of f by verifying both compositions

$$f^{-1} \circ f \quad \text{and} \quad f \circ f^{-1}.$$

Conclude that f is a bijection.