MCS 441 – Theory of Computation I
Spring 2013
Problem Set 1

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Due: 1/23/13 at the beginning of class

Related reading: Chapter 0
Instructions: Atop your problem set, please write your name, clearly list your collaborators (see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student.
Important note: Problems labeled "U" and "G" are assigned to undergraduate and graduate students, respectively. Undergraduate students can get a small bonus for solving the graduate problems. Graduate students are encouraged to solve the undergraduate problems for practice.

Set Theory

1. [**6 pts**] For each pair of sets A and B, are they the same? Briefly explain.

i. $A \equiv \{\},\ B \equiv \{\emptyset\}$

ii. $A \equiv \mathcal{Z},\ B \equiv \mathcal{Z} \cup \mathcal{N}$

iii. $A \equiv \{i : i = m^2 \text{ for some } m \in \mathcal{N}\},\ B \equiv \mathcal{N} \times \mathcal{N}$

Boolean Logic and Functions

2. [**6 pts**] Two boolean expressions are equivalent if and only if their truth tables are the same. How many *inequivalent* boolean expressions exist over 3 variables? Explain your answer.

Induction

3. [**6 pts**] Prove by induction on k:

U. Prove that for all integers $k \geq 4,\ k! > 2^k$.

G. Assume that c is a real number such that $c + \frac{1}{c}$ is an integer. Prove that for all integers $k > 0$,

$$c^k + \frac{1}{c^k}$$

is an integer.

Strings and Sequences

4. [**6 pts**] Imagine the following binary sequence: begin with the number 0 and continually append the Boolean negation of the sequence so far. Hence, the sequence begins as

$$0110100110010110...$$

One can think of the elements, t_i, of the sequence individually, starting with t_0. So, we have

$$t_0 = 0, t_1 = 1, t_2 = 1, t_3 = 0, t_4 = 1, t_5 = 0, ...$$

i. What is the value of t_{256}? How did you arrive at you answer?

ii. Does the string "111" occur (as a contiguous substring) somewhere in this sequence? If so, where is the first occurrence? If not, why not?

Graphs

5. [**6 pts**] Prove that every graph on $n \geq 2$ vertices (and no self-loops) has at least two vertices of the same degree.