MCS 441 – Theory of Computation I
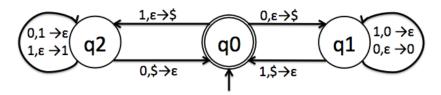## Spring 2013
Problem Set 5

Lev Reyzin

Due: 2/27/13 at the beginning of class

Related reading: Chapters 2.1-2.3

Instructions: Atop your problem set, write your name, clearly list your collaborators[1] (see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student. Answer all questions in the order they are assigned.

1. **[10 pts]** Let PDA P_1 be:

   a. [2 pts] Give a formal description of P_1. (Be careful in this part because points will be deducted for small mistakes.)

   b. [3 pts] What language does P_1 recognize?

   c. [3 pts] Give a CFG that generates $L(P_1)$.

   d. [2 pts] Is $L(P_1)$ regular? Why or why not?

2. **[10 pts]** Consider the CFG G_2:

$$
\begin{aligned}
S &\rightarrow 0X \\
X &\rightarrow 0X \\
X &\rightarrow 1X \\
X &\rightarrow 1
\end{aligned}
$$

   a. [3 pts] What language does G_2 generate?

   b. [2 pts] Is $L(G_2)$ regular? Why or why not?

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[1]If you did not have any collaborators, please say so.

c. [3 pts] Give a CFG in CNF generating $L(G_2)$.

d. [2 pts] Is your CFG in [c.] ambiguous? Why or why not?

3. [10 pts] Consider the following languages over $\Sigma = \{0, 1\}$:

$$
\begin{aligned}
L_{3.1} &= \{0^n 1^n 0^n 1^n \mid n \geq 0\} \\
L_{3.2} &= \{st \mid s, t \in \Sigma^*, s \neq t, |s| = |t|\}
\end{aligned}
$$

a. [5 pts] Is $L_{3.1}$ context free? Show your answer is correct.

b. [5 pts] Is $L_{3.2}$ context free? Show your answer is correct.

4. [5 pts] Prove that if A is a context-free language, then so is $A^{\leftrightarrow}$. Refer to Problem Set 3 for our definition of $\leftrightarrow$.