MCS 441 – Theory of Computation I
Spring 2013
Problem Set 8

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Due: 4/5/13 at the beginning of class

Related reading: Chapters 5 and 6.
Instructions: Atop your problem set, write your name, list your collaborators (see syllabus for the collaboration policy), and indicate whether you are an undergraduate or graduate student.

Rice's Theorem

1. [9 pts] Consider the following theorem.

Theorem 1 (Rice '53) *Let P be a language consisting of Turing machine descriptions where P fulfills the following two conditions:*

1. *P contains some but not all TM descriptions.*
2. *For any two TMs M_1 and M_2, whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$.*

Then P is undecidable.

Solve the following two problems.

a. [3 pts] Invent an interesting undecidable language and show it is undecidable using Theorem 1.

b. [6 pts] Show that both conditions *1.* and *2.* of Theorem 1 are necessary for it to be true.

Post Correspondence Problem

2. [6 pts] Consider the Post Correspondence Problem (PCP). In Sipser's example on p. 227 ch. 5.2, the strings written on the dominos are over the alphabet $\{a, b, c\}$. That variant of PCP is undecidable. Now consider a different variant of the problem where the alphabet is simply $\{a\}$, i.e. the strings on the dominos are composed of a's. Prove this latter variant of PCP is decidable.[1]

Many-to-one and Turing reductions

3. [8 pts] Let $\Sigma = \{0, 1\}$ and $L \subseteq \Sigma^*$.

a. [4 pts] Show that L is decidable if and only if $L \leq_T \Sigma^*$.

b. [4 pts] Is the statement above true if we replace $\leq_T$ with $\leq_m$? Why or why not?

[1] To solve this problem, it is not necessary to understand the proof of Theorem 5.15

Kolmogorov complexity

4. [11 pts] Let $K(x)$ be the Kolmogorov complexity (or "descriptive complexity") of string x.

a. [6 pts] Show that an oracle Turing machine with an oracle for

$$\text{HALT}_{\text{TM}} = \{\langle M, w\rangle \mid M \text{ is a Turing machine that halts on } w\}$$

can compute $K(x)$. Formalize this problem as needed.

b. [5 pts] For $i \geq 1$, give the best upper bound you can for $K(x^i)$ as a function of $K(x)$ and i.