MCS 541 – Computational Complexity
Spring 2023
Problem Set 3*

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Due: 3/6/23 at the beginning of class

1. Give a proof of Theorem 4.8 from Arora-Barak (Space Hierarchy Theorem). You may use the theorem's proof from Sipser's book for guidance, but your write-up should use the logic and notation of Theorem 3.1 from Arora-Barak (Time Hierarchy Theorem).

2. Show that if we change Definition 4.19 in Arora-Barak (verifier definition of **NL**) to allow the certificate to be read on a usual real-only tape (instead of a read-once tape), then the resulting class that is defined is equal to **NP**.

3. Give an example of a function $T : \mathbb{N} \to \mathbb{N}, T(n) \geq n$ that is not time-constructible. Your example should have the property that T bounded from above by some other function that *is* time-constructible. Prove your answer correct.

4. Show that TQBF remains complete for **PSPACE** under logspace reductions.

5. Show that the problem of determining whether a given nondeterministic finite automaton (NFA) accepts a given string is complete for **NL**.

*Most of these problems are modifications of exercises that appear in Arora-Barak.