MCS 541 – Computational Complexity
Spring 2023
Problem Set 5*

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Due: 4/17/23 at the beginning of class

1. Find a *decidable* language in $\mathbf{P_{/poly}} \setminus \mathbf{P}$. (Recall that UHALT is an undecidable language in $\mathbf{P_{/poly}} \setminus \mathbf{P}$.)

2. In class we saw that a PTM using unbiased coins can efficiently simulate a PTM using coins of bias ρ. This proof, however, needed us to (efficiently) compute ρ. Prove that if the coin's bias ρ is allowed to be arbitrary, then a PTM using coins of bias ρ can decide (with, say, two-sided error) some undecidable language in polynomial time.

3. It is trivial to prove that every function f from $\{0,1\}^n$ to $\{0,1\}$ can be computed by a Boolean circuit of size $O(n2^n)$. Improve this bound to $O(2^n)$.

4. Define **EPP** to be the class of languages decided by probabilistic Turing machines that give the correct accept/reject answer on any input x with probability $> 1/2$ (similar to the relaxed requirement of **BPP** compared to the stringent requirement for **ZPP**), but whose *expected* running time is polynomial (similar to the relaxed requirement for **ZPP** compared to the stringent requirement for **BPP**). Show that $\mathbf{EXP} \subseteq \mathbf{EPP}$.

*Many of these problems are modifications of exercises that appear in Arora-Barak.