Metamathematics I
Problem Set 4

Due Friday October 12

1) We say that a group $(G, +, <)$ is *archimedean* if for all $x, y \in G$ with $x, y > 0$ there is an integer m such that $|x| < m|y|$. Show that there are non-archimedean fields elementarily equivalent to the field of real numbers.

2) Show that there is no formula $\phi(v)$ in the language of groups such that for all groups G if $g \in G$, $G \models \phi(g)$ if and only if g has finite order.

3) Let Ł be the language with one binary relation symbol $<$. Let T be an Ł-theory extending the theory of linear orders such that T has infinite models. Show that there is $\mathcal{M} \models T$ and an order preserving embedding $\sigma : \mathbf{Q} \to \mathcal{M}$ of the rational numbers into M. [Hint: Use the Compactness Theorem.]

For example if T is the full theory of the $(\mathbf{Z}, <)$, there is $\mathcal{M} \equiv \mathbf{Z}$ in which the rational order embedds.

4) Show that every torsion free abelian group $(G, +)$ can be linearly ordered such that $(a < b \wedge c \leq d) \to a + c < b + d$. [Hint: First show this for finitely generated groups. Then use compactness.]