Errata to: Tree of tuples of a structure

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May 2026

We correct the proof of Corollary 1.11 of [HTM22] as follows.

Corollary 1.11. *For each computable ordinal α, there is a structure $\mathcal{A}$ with no $0^{(\alpha)}$-computable copies but for which $\mathcal{T}(\mathcal{A})$ has a computable copy.*

Proof. We may choose a successor ordinal β large enough that $\alpha + \beta = \beta$. Relativize Theorem 1.10 to $0^{(\beta)}$ to obtain a structure $\mathcal{A}$ be such that $\mathcal{T}(\mathcal{A})$ has a $0^{(\beta)}$-computable copy but $\mathcal{A}$ has no $0^{(\beta)}$-computable copy. Let $\mathcal{B}$ be $\mathcal{A}^{(-\beta)}$. Then by Lemma 6.3 $\mathcal{T}(\mathcal{B})$ has a computable copy. Because $\alpha + \beta = \beta$, $\mathcal{B}$ has no $0^{(\alpha)}$-computable copy; if it did, then by Lemma 6.2 relativized to $0^{(\alpha)}$, $\mathcal{A}$ would have a $0^{(\alpha+\beta)} = 0^{(\beta)}$-computable copy. □

References

[HTM22] Matthew Harrison-Trainor and Antonio Montalbán. The tree of tuples of a structure. *J. Symb. Log.*, 87(1):21–46, 2022.