Math 180 Worksheet 1 Partial Solutions

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Problem (1-d)**.** The simplest example of a function f defined on the real numbers and satisfying $f(x) \geq 0$ for every x and $f(-4) = -f(4)$, is probably the zero function:

$$f(x) = 0 \text{ for all real } x.$$

However, we can think of other (non-zero) functions that we can determine, and a possible way of obtaining some examples is as follows:

So, we want the function f to satisfy both $f(x) \geq 0$ for all x and $f(4) = -f(-4)$. Suppose $f(4) > 0$. Then, $f(-4) = -f(4)$ is a negative number, contradicting the second condition. This means that $f(4) \leq 0$. We already know from the second condition that $f(4) \geq 0$, and this gives us $f(4) = 0$. Thus any function f that we come up with, has to satisfy $f(4) = f(-4) = 0$.

Now comes the main part. there are many functions that have roots at 4 and -4, the easiest one being $f(x) = x^2 - 16$. The problem with this is that it falls below the x-axis at points in $(-4, 4)$, so one possible way to overcome this is by taking the absolute value! So we have an example

$$f(x) = |x^2 - 16| \text{ for all real } x.$$

Some other functions that can work are,

$$f(x) = |\sin 4x| \text{ for all real } x,$$

or

$$f(x) = (x^2 - 16)^{48} \text{ for all real } x,$$

etc. □

Problem (3-c)**.** Well, the only thing to keep in mind is that the value of sin at integer multiples of π is zero. The rest follows from plugging into the formula for slope:

$$\begin{aligned}\text{Slope between } (0, f(0)) \text{ and } (k\pi, f(k\pi)) &= \frac{f(k\pi) - f(0)}{k\pi - 0} \\ &= \frac{\sin(k\pi) - \sin 0}{k\pi - 0} \\ &= \frac{0}{k\pi} = 0.\end{aligned}$$

□

Problem (3-d)**.** I got a few wild stares for this. The problem asked for pairs other than the pairs 0 and $k\pi$ which have slope zero. From the graph in part (a) (which I am too lazy to draw), we can see that there are several possibilities for secant lines that are parallel to the x axis. Note that any secant line parallel to the x axis has a slope of 0.

And examples of such points are,

- $(\pi, f(\pi))$ and $(2\pi, f(2\pi))$
- $(\pi, f(\pi))$ and $(2016\pi, f(2016\pi))$
- $\left(\frac{\pi}{2}, f\left(\frac{\pi}{2}\right)\right)$ and $\left(\frac{5\pi}{2}, f\left(\frac{5\pi}{2}\right)\right)$

$\vdots$

$\square$

Food for thought. For the last set of answers in $3-d$, the secant line joining the points on the graph of sin with values at $\frac{\pi}{2}$ and $\frac{5\pi}{2}$ is parallel to the x-axis. That means $f(\frac{\pi}{2}) = f(\frac{5\pi}{2})$. What is the difference between these two x-values? Is there anything special about the difference of 2π?