Math 170: Quiz 14

Sayan Mukherjee's discussion

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Problem 1. Consider $f(x) = x^3 - 6x^2 - 15x + 1$ on the interval $[-2, 6]$. Find its intervals of monotonicity and local extrema.

Solution. The derivative of f is

$$f'(x) = 3x^2 - 12x - 15 = 3(x^2 - 4x - 5) = 3(x - 5)(x + 1).$$

Hence $f'(x) = 0$ at $x = -1$ and $x = 5$, and these are our critical points in the interval $[-2, 6]$.

We need to find the intervals of monotonicity, so plug in values in the intervals $[-2, -1), (-1, 5), (5, 6]$ to figure out what happens to f'.

$f'(-2) = 3(-7)(-1) > 0$, $f'(0) = 3(-5)(1) < 0$, $f'(6) = 3(1)(7) > 0$.

Thus, f is increasing on the intervals $[-2, -1]$ and $[5, 6]$, and decreasing on the interval $[-1, 5]$.

Additionally, to figure out which extrema is a maximum and which is a minimum, we can see what happens to the left and right of the critical points. Since f is increasing to the left of -1 and decreasing to the right, it looks like a crown at -1, meaning that -1 is a maximum. Similarly, f is decreasing to the left of 5 and increasing to the right, so 5 is a minimum.

Local max: $x = -1$ and local min: $x = 5$. $\square$

Rubric.

- Finding $f'(x)$: +2 pts
- Finding roots $x = 5$ and $x = -1$: +1pt
- Stating that $x = 5$ and $x = -1$ are the extrema (no need to classify them as min/max): +1pt
- Determining that f is increasing on $[-2, -1] \cup [5, 6]$ and decreasing on $[-1, 5]$: +1pt