

# Math 170: Quiz 15 Solution

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March 16, 2021

**Problem 1.** Use the second derivative test to figure out and classify the local extrema of

$$f(x) = x^4 - 8x^2.$$

*Solution.* The critical points of this function are obtained by solving  $f'(x) = 0$ . Observe that,

$$f'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x - 2)(x + 2),$$

And therefore the roots of  $f'(x) = 0$  are  $x = 0, 2, -2$ . We need to check which of these is a max and which is a min using the second derivative test. Note that  $f'(x) = 4x^3 - 16x$  implies that  $f''(x) = 12x^2 - 16$ .

- $x = -2$ :  $f''(-2) = 12(-2)^2 - 16 = 48 - 16 = 36 > 0$ , so  $f$  looks like a “cup” at  $-2$ , and it's a local min
- $x = 0$ :  $f''(0) = 12(0)^2 - 16 = -16 < 0$ , so  $f$  looks like a “crown” at  $0$ , and it's a local max
- $x = 2$ :  $f''(2) = 12(2)^2 - 16 = 48 - 16 = 36 > 0$ , so  $f$  looks like a “cup” at  $2$ , and it's a local min.

□

*Rubric.*

- Left to the grader's discretion.