Math 170: Quiz 16 Solution

Sayan Mukherjee's discussion

March 18, 2021

Problem 1. Use the **Newton-Raphson approximation method** in order to find a recursive formula that approximates the fourth root of 17.

Solution. We're only asked to find the recursive formula. As we're computing the fourth root, we will use $f(x) = x^4 - 17$, then $f'(x) = 4x^3$. The recursive formula then becomes,

$$
\begin{aligned}
x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \\
&= x_n - \frac{x_n^4 - 17}{4x_n^3} \\
&= x_n - \frac{x_n^4}{4x_n^3} + \frac{17}{4x_n^3} \\
&= \frac{3x_n}{4} + \frac{17}{4x_n^3}.
\end{aligned}
$$

Therefore the recursion is $x_{n+1} = \frac{3x_n}{4} + \frac{17}{4x_n^3}$. $\qquad\square$

Rubric.

- Left to the grader's discretion.