Math 170: Quiz 18

Sayan Mukherjee's discussion

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Problem 1. Let $\zeta(s)$ be the function defined as follows:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

It is known that $\zeta(s)$ is finite when s is a real number less than 1, implying that ζ is meromorphic, and hence can be analytically continued to a function valid for all complex s.

Prove or disprove the following statement:

$\zeta(s)$ has only zeros at the negative even integers and complex numbers with real part $\frac{1}{2}$.