On the interval $[0,1]$, note that e^{x^3} is increasing, hence

$$e^{0^3} \leq e^{x^3} \leq e^{1^3}$$

$$\Rightarrow \quad 1 \leq e^{x^3} \leq e$$

Hence, we have

$$1 \cdot (1-0) \leq \int_0^1 e^{x^3} \leq e \cdot (1-0) \text{ , implying}$$

$$1 \leq \int_0^1 e^{x^3} \leq e \text{ , as desired.}$$