

Math 170: Quiz 3

Sayan Mukherjee's discussion

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Problem 1. Show using the ϵ -definition of limits, that

$$\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \frac{1}{2}.$$

Solution. Fix $\epsilon > 0$. We wish to solve for n in the following inequality:

$$\left| \frac{n}{2n+1} - \frac{1}{2} \right| < \epsilon.$$

This is equivalent to

$$\left| \frac{2n - (2n+1)}{2(2n+1)} \right| < \epsilon,$$

Or,

$$\left| \frac{-1}{4n+2} \right| < \epsilon.$$

As $|-1| = 1$, we gotta solve

$$\frac{1}{4n+2} < \epsilon,$$

Which gives us

$$\frac{4n+2}{1} > \frac{1}{\epsilon} \iff 4n > \frac{1}{\epsilon} - 2 \iff n > \frac{1}{4} \left(\frac{1}{\epsilon} - 2 \right).$$

□

Rubric.

- +2 for correct equation $\left| \frac{n}{2n+1} - \frac{1}{2} \right| < \epsilon$
- +2 for attempted algebra (even if incorrect due to sign errors or reciprocation errors).
- +1 for correct expression $n > \frac{1}{4\epsilon} - \frac{1}{2}$ or anything equivalent to this expression.