

# Math 170: Worksheet 5 (solution sketches)

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## Problem 1.

1.  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 + 1} = 0.$

2.  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^5 - 1} = \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x^4 + x^3 + x^2 + x + 1} = \frac{3}{5}.$

3.  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^3 - x^2 - x - 2} = \lim_{x \rightarrow 2} \frac{x^2 + 2x + 4}{x^2 + x + 1} = \frac{12}{7}.$

4. We multiply both numerator and denominator by their conjugates for this problem:

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{\sqrt{x+1} - \sqrt{3}} &= \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x+2} + 2} \cdot \frac{\sqrt{x+1} + \sqrt{3}}{x-2} \\ &= \lim_{x \rightarrow 2} \frac{\sqrt{x+1} + \sqrt{3}}{\sqrt{x+2} + 2} \\ &= \frac{\sqrt{3} + \sqrt{3}}{\sqrt{4} + 2} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}. \end{aligned}$$

5. Similar to above:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5} - 2}{\sqrt{x+1} - 2} &= \lim_{x \rightarrow 3} \frac{x^2 - 5 - 4}{\sqrt{x^2 - 5} + 2} \cdot \frac{\sqrt{x+1} + 2}{x+1-4} \\ &= \lim_{x \rightarrow 3} \frac{x^2 - 9}{\sqrt{x^2 - 5} + 2} \cdot \frac{\sqrt{x+1} + 2}{x-3} \\ &= \lim_{x \rightarrow 3} \frac{(x+3)(x-3)(\sqrt{x+1} + 2)}{(\sqrt{x^2 - 5} + 2)(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{(x+3)(\sqrt{x+1} + 2)}{(\sqrt{x^2 - 5} + 2)} \\ &= \frac{6 \cdot 4}{4} = 6. \end{aligned}$$