Quasiconvex Subgroups of Acylindrically Hyperbolic Groups

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Hyperbolic Groups

Hyperbolic group

A group G is hyperbolic if for some finite $X \subseteq G$, the Cayley graph $\Gamma(G, X)$ is connected and δ–hyperbolic for some $\delta \geq 0$.

Quasiconvex subgroup

A subgroup H of G is quasiconvex if the inclusion map is a quasi-isometric embedding.

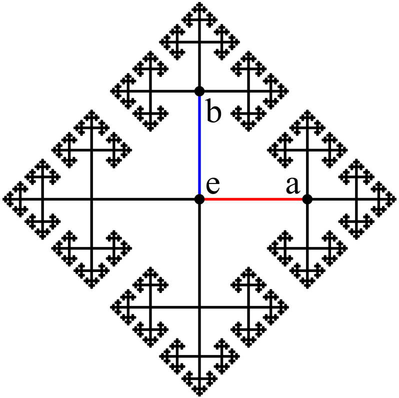

Figure: Cayley graph of $\mathbb{F}^2$
[Wikimedia Commons contributors, 2024]

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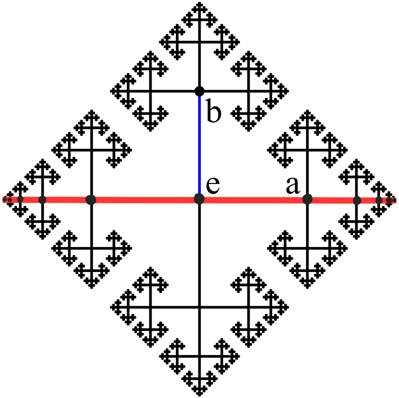

Figure: $\mathbb{Z}$ is a quasiconvex subgroup of $\mathbb{F}^2$

Acylindrically Hyperbolic Groups

Acylindrically hyperbolic group [Martínez-Pedroza and Rashid, 2021]

A group G is acylindrically hyperbolic if for some $X \subseteq G$ and some finite collection $\mathcal{P}$ of infinite subgroups, the coned-off Cayley graph $\hat{\Gamma}(G, \mathcal{P}, X)$ is connected, hyperbolic and fine at cone vertices.

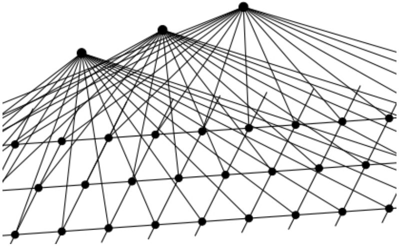

Figure: Coned-off Cayley graph of $\mathbb{Z} \oplus \mathbb{Z}$ along $\mathbb{Z}$ [Page, 2017]

$(G, \mathcal{P})$-Quasiconvex Subgroups

Acylindrically hyperbolic group [Martínez-Pedroza and Rashid, 2021]

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$(G, \mathcal{P})$-quasiconvex subgroup [Wan]

A subgroup H is $(G, \mathcal{P})$-quasi-convex, if there exists a compatible $\mathcal{D}$, and the extension of inclusion $\hat{\Gamma}(H, \mathcal{D}, Y) \hookrightarrow \hat{\Gamma}(G, \mathcal{P}, X)$ is a quasi-isometric embedding.

Application

Dehn Filling

Given a collection of subgroups $N_\lambda \lhd P_\lambda$, let K be the normal closure of $\cup_{\lambda\in\Lambda} N_\lambda$ in G. The *filling* of G is the quotient group $\bar{G} = G/K$.

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Theorem (Wan)

Suppose H is $(G, \mathcal{P})$-quasi-convex. For sufficiently long H-fillings of G with kernel K, the induced filling on H has kernel $K_H = K \cap H$.
Moreover, let $\bar{\mathcal{P}} = \{P_\lambda / N_\lambda\}$, let $\bar{H} = H/K_H$. Then $\bar{H}$ is $(\bar{G}, \bar{P})$-quasi-convex.

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Theorem (Wan)

Suppose G acts cocompactly on a CAT(0) *cube complex X. Suppose every parabolic element of G fixes some point of X and that cell-stabilizers are full $(G, \mathcal{P})$-quasi-convex. For sufficiently long fillings of G that fix each G-orbits of cubes of X pointwise, the quotient X/K is a* CAT(0) *cube complex.*

Thank You

References

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