Math 165 / Lowman **Quiz 3 (v4)** **November 16, 2010**

TA Ali Mohajer

SOLUTIONS by Ali Mohajer

1. Write the general power, exponential and log rules for both differentiation and integration.

   SOLUTION:

   $$\frac{d}{dx}(f(x))^n = nf'(x)(f(x))^{n-1}$$ general power rule for differentiation

   $$\int f'(x)(f(x))^n dx = \frac{(f(x))^{n+1}}{n+1} + C$$ general power rule for integration

   $$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$$ general exponential rule for differentiation

   $$\int f'(x)e^{f(x)}dx = e^{f(x)} + C$$ general exponential rule for integration

   $$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$$ general log rule for differentiation

   $$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$$ general log rule for integration

2. Use logarithmic differentiation to find $\frac{df}{dx}$ where $f(x) = (9x+1)^x$.

SOLUTION:

Taking the natural logarithm of both sides of the equation

$$f(x) = (9x+1)^x$$

yields:

$$\ln(f(x)) = \ln((9x+1)^x)$$

now recall that $\ln(a^b) = b\ln(a)$, so we can re-write the right-hand side as shown below:

$$\ln(f(x)) = x\ln(9x+1)$$

now we differentiate both sides of the above to get:

$$\frac{d}{dx}\ln(f(x)) = \frac{d}{dx}(x\ln(9x+1))$$

now on the left-hand side we apply the general log rule for differentiation, and on the right hand side we apply the product rule for differentiation to get:

$$\frac{f'(x)}{f(x)} = (\frac{d}{dx}x)\ln(9x+1) + x(\frac{d}{dx}\ln(9x+1))$$

now on the right hand side we evaluate the derivatives to get:

$$\frac{f'(x)}{f(x)} = \ln(9x+1) + x\frac{9}{9x+1}$$

and finally we multiply both sides by $f(x)$ (which is $(9x+1)^x$) to get our final result:

$$f'(x) = (\ln(9x+1) + \frac{9x}{9x+1})(9x+1)^x.$$

3. Evaluate the three definite integrals below.
(Use the general power rule, exponential rule or log rule. Do not use the substitution method.)

(a) $\int_3^5 2x(x^2+1)^6 dx$

SOLUTION:

We apply the general power rule for integration from question (1), noting that we are choosing $f(x) = x^2 + 1$, and that $f'(x) = 2x$ is already present in the integrand, so there is no need to apply a "multiply by one trick" to obtain the differential. With this in mind, it is clear that:

$$\int_3^5 2x(x^2+1)^6 dx = \left[\frac{(x^2+1)^7}{7}\right]_3^5 = \frac{((5)^2+1)^7}{7} - \frac{((3)^2+1)^7}{7} = \frac{26^7 - 10^7}{7}$$

(b) $\int_3^5 3e^{3x+1} dx$

SOLUTION:

We apply the general exponential rule for integration from question (1), noting that we are choosing $f(x) = 3x+1$, and that $f'(x) = 3$ is already present in the integrand, so once agian there is no need to apply a "multiply by one trick" to obtain the differential. With this in mind, it is clear that:

$$\int_3^5 3e^{3x+1} dx = \left[e^{3x+1}\right]_3^5 = e^{3(5)+1} - e^{3(3)+1} = e^{16} - e^{10}$$

(c) $\int_3^5 \frac{1}{6x+1} dx$

SOLUTION:

We apply the general log rule for integration from question (1), noting that we are choosing $f(x) = 6x + 1$. In this case however, we note that $f'(x) = 6$ is not present in the integrand, so we do need to apply a "multiply by one trick" to obtain the differential form $6dx$:

$$\int_3^5 \frac{1}{6x+1} dx = \int_3^5 \frac{\frac{1}{6}6}{6x+1} dx = \frac{1}{6}\int_3^5 \frac{6}{6x+1} dx$$

Now it is clear that the right-most integral in the line above is:

$$= \frac{1}{6}\left[\ln|6x+1|\right]_3^5 = \frac{1}{6}(\ln|6(5)+1| - \ln|6(3)+1|) = \frac{1}{6}(\ln(31) - \ln(19)) = \frac{1}{6}(\ln(\frac{31}{19}))$$