10. $f(x)$, $f'(x) = -10x^2 + 60x - 50 = 0$

$= -10(x^2 - 6x + 5) = 0$

$= -10\underbrace{(x - 5)}_{0}\underbrace{(x - 1)}_{0} = 0$

$x_c = 5, 1$

$f''(x) = -10(2x - 6) = -20(x-3)$
$= 20(3 - x)$

$f''(5) = 20(3-5) = (-)$ max at $x=5$

$f''(1) = 20(3-1) = (+)_{H_2O}$ min at $x=1$

11. See posted solutions for other exams.

12

14.

13. $f' = (2x+7)(5x^2+x)^{1/2}$

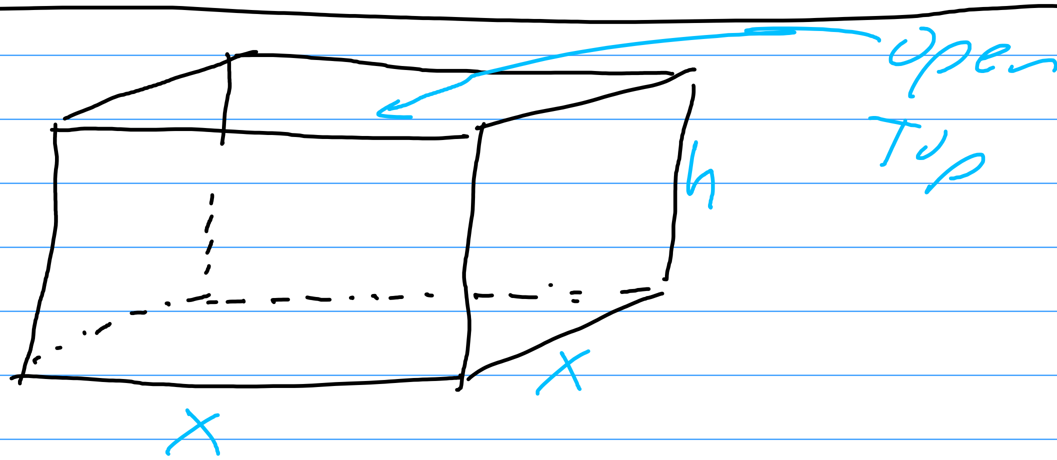
$+ (x^2 + 7x + 1)\frac{1}{2}(5x^2+x)^{-\frac{1}{2}} \cdot (10x+1)$

at $x=1$

$$f'(1) = (2(1)+7)(5(1)^2+1)^{1/2}$$
$$+(1^2+7(1)+1)\cdot\frac{1}{2}\frac{(10(1)+1)}{\sqrt{5(1)^2+1}}$$

$$= 9\sqrt{6} + \frac{9(11)}{2\sqrt{6}}$$
$$= 9\sqrt{6} + \frac{99}{2\sqrt{6}}$$

15.



Bottom has Area $A_b = x^2$

Each side has Area $A_s = xh$

total area:

$$A = 4\cdot A_s + A_b = x^2 + 4xh$$

$$\text{Volume} = x^2 h = 7\ m^3$$

Minimize Area subject to

Volume Constraint:

Minimize $A = x^2 + 4xh$

S.T. $x^2 h = 7$

$h = \frac{7}{x^2}$

$A(x) = x^2 + 4x\left(\frac{7}{x^2}\right) = x^2 + \frac{28}{x}$

$= x^2 + 28x^{-1}$

$A'(x) = 2x - 28x^{-2}$

$A''(x) = 2 + 56x^{-3}$

$A'(x) = \cancel{2}x - \frac{\cancel{28}\,14}{x^2} = 0$

$\cancel{x}^3 - 14 = 0$

$x = \sqrt[3]{14} \cong 14^{1/3} \cong 2.41m$

$h = \frac{7}{x^2} = \frac{7}{(14)^{2/3}} = 1.21m$

---

min Area

$A = x^2 + \frac{28}{x} = 14^{2/3} + \frac{28}{14^{1/3}}$

$\cong \boxed{17.43\ m^2}$ Minimum Area that will hold $7m^3$

16. $C(x) = \frac{x^2}{5} + 6x + 1000$

Average Cost:

$$\bar{C} = \frac{C(x)}{x} = \frac{x}{5} + 6 + \frac{1000}{x}$$

$$\bar{C}' = \frac{1}{5} + 0 - \frac{1000}{x^2} = 0$$

$$x^2 = 5000$$

$$x = \sqrt{5000} = \boxed{\$70.71}$$

Use 2nd derivative test to check if minimum.

$$\bar{C}''(x) = +\frac{2000}{x^3}$$

$$\bar{C}''(70.71) = +\frac{2000}{(70.71)} = (+)$$

min ✓

17. $3xy^2 + y = 10 \quad , \; y = y(x)$

$$\frac{d}{dx}(3xy^2 + y) = \frac{d}{dx}10$$

$$\frac{d}{dx}(3x) \cdot y^2 + 3x \cdot \frac{d}{dx}y^2 + \frac{dy}{dx} = 0$$

$$3y^2 + 3x \cdot 2y \cdot y' + y' = 0$$

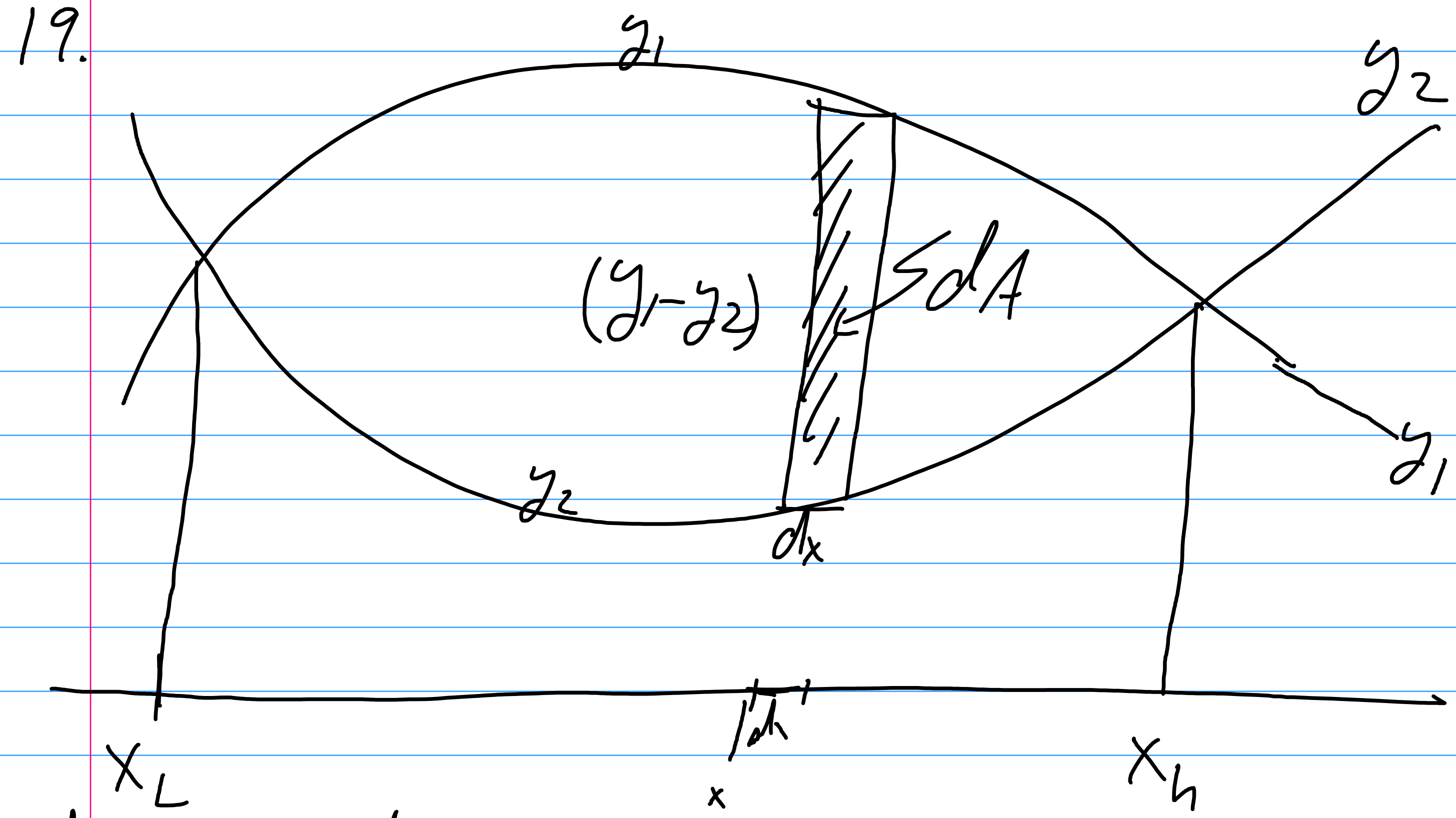
$$3y^2 + 6xy \cdot y' + y' = 0$$

$$y' \cdot (6xy + 1) = -3y^2$$

$$\boxed{y'(x) = \frac{-3y^2}{(6xy+1)}} \quad \text{at } (x,y) = (2,1)$$

$$y'(2) = \frac{-3(1)^2}{6(2)(1)+1} = \boxed{\frac{-3}{13}}$$

19.



$$dA = (y_1 - y_2)dx$$

$$= [(-4x^2 + 20x - 12) - (4x^2 - 20x + 20]dx$$

$$dA = (-8x^2 + 40x - 32)dx$$

Limits: Solve $y_1 = y_2$

$$-4x^2 + 20x - 12 = 4x^2 - 2x + 20$$

$$8x^2 - 40x + 32 = 0$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x = 4, 1$$

$$\Rightarrow x_L = 1, \quad x_H = 4$$

$$A = \int_{x=1}^{4} (-8x^2 + 40x - 32)\,dx$$

$$= \left[-\frac{8x^3}{3} + \frac{40x^2}{2} - 32x \right]_1^4$$

$$= \underline{\hspace{6cm}}$$

finish the calculation