#5 $f(x) = \dfrac{\sqrt{x+1} - 1}{x}$

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x} \overset{try}{=} \frac{\sqrt{0+1} - 1}{0} \to \frac{0}{0} \text{ indeterminant form}$$

try Standard Trick:

$$= \lim_{x \to 0} \left(\frac{\sqrt{x+1} - 1}{x} \right) \left(\frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \right)$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{x+1}\right)^2 - 1^2}{x\left(\sqrt{x+1} + 1\right)} = \lim_{x \to 0} \left[\frac{x+1 - 1}{x\left(\sqrt{x+1} + 1\right)} \right]$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+1} + 1\right)} = \lim_{x \to 0} \frac{1}{\sqrt{x+1} + 1} = \frac{1}{\sqrt{0+1} + 1}$$

$$= \boxed{\frac{1}{2}}$$

If the limit of $\dfrac{f(x)}{g(x)}$ gives either $\dfrac{0}{0}$ OR $\dfrac{\infty}{\infty}$

then you can use L'Hôpitals Rule:
(This is given in an Appendix of the textbook)

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$ This only works for Indeterminate forms

$$\Rightarrow \lim_{x \to 0} \underbrace{\left[\frac{\sqrt{x+1} - 1}{x} \right]}_{= \frac{0}{0}} \overset{\gg}{=} \lim_{x \to 0} \left[\frac{\frac{d}{dx}(x+1)^{1/2} - 1}{\frac{d}{dx} x} \right] = \lim_{x \to 0} \frac{\frac{1}{2}(x+1)^{-1/2} - 0}{1}$$

$$= \lim_{x \to 0} \frac{1}{2\sqrt{x+1}} = \frac{1}{2\sqrt{0+1}} = \boxed{\frac{1}{2}}$$