

$$16. \int u'(x) \cdot u(x)^n dx = \frac{u(x)^{n+1}}{n+1} + C \quad \text{Power}$$

$$17. \int u'(x) \cdot e^{u(x)} dx = e^{u(x)} + C \quad \text{Exponential}$$

$$\int \frac{u'(x)}{u(x)} dx = \ln |u(x)| + C \quad \text{Log Rule.}$$

$$17. I = \int (x^6 + x^2) (3x^7 + 7x^3)^{10} dx$$

$$u(x) = 3x^7 + 7x^3 \quad u'(x) = 3 \cdot 7x^6 + 7 \cdot 3x^2$$

$$= 3 \cdot 7 (x^6 + x^2) = 21(x^6 + x^2)$$

$$I = \frac{1}{21} \int 21 (x^6 + x^2) \cdot (3x^7 + 7x^3)^{10} dx$$

$$= \frac{1}{21} \cdot \frac{(3x^7 + 7x^3)^{11}}{11} = \boxed{\frac{(3x^7 + 7x^3)^{11}}{231}} + C$$

$$18. I = \int 21 (x^6 + x^2) e^{(3x^7 + 7x^3)} dx$$

$$u(x) = 3x^7 + 7x^3 \quad u'(x) = 21(x^6 + x^2) \Rightarrow \text{perfect fit}$$

$$\boxed{I = e^{3x^7 + 7x^3}}$$

$$19. \int \frac{21(x^6 + x^2)}{(3x^7 + 7x^3)} dx = \boxed{\ln |3x^7 + 7x^3| + C}$$