

3. $f(x) = (2+3x)^x$

Note, this is neither an exponential function nor a power function because there are variables in both the base and the exponent.

Use logarithmic differentiation

$$\ln f = \ln[(2+3x)^x]$$

$$\ln f = x \cdot \ln(2+3x)$$

$$\frac{d}{dx} \ln f = \frac{d}{dx} [x \cdot \ln(2+3x)]$$

log rule
↓

product rule
↓

$$\frac{f'(x)}{f(x)} = \frac{d}{dx} x \cdot \ln(2+3x) + x \cdot \frac{d}{dx} \ln(2+3x)$$

$$\frac{f'(x)}{f(x)} = \ln(2+3x) + x \cdot \frac{\frac{d}{dx}(2+3x)^3}{2+3x}$$

$$\frac{f'(x)}{f(x)} = \ln(2+3x) + x \cdot \frac{3}{2+3x}$$

Replace
↓

$$f'(x) = f(x) \cdot \left[\ln(2+3x) + \frac{3x}{2+3x} \right]$$

$$f'(x) = (2+3x)^x \cdot \left[\ln(2+3x) + \frac{3x}{2+3x} \right] \quad (5)$$