

10. $f(x) = 1 + \frac{x}{(1+x)^2} \quad \left| \quad f'(x) = \frac{1-x}{(1+x)^3} \quad \right| \quad f''(x) = \frac{2(x-2)}{(1+x)^4}$

$$f'(x) = 0 \Rightarrow \frac{1-x}{(1+x)^3} = 0 \Rightarrow 1-x = 0 \Rightarrow x = 1$$

$$f''(x) = \frac{2(x-2)}{(1+x)^4} \Rightarrow f''(1) = \frac{2(1-2)}{(1+1)^4} = \frac{-2}{16} = -\frac{1}{8}$$

10.1 $f(x)$ is a rational function (can put over common denominator)

10.2 y-int: $(0, ?)$ $y = f(0) = 1 + \frac{0}{(1+0)^2} = 1 \Rightarrow$ y-int: $(0, 1)$

10.3 x-ints: $(?, 0)$ $0 = 1 + \frac{x}{(1+x)^2} \Rightarrow \frac{(1+x)^2 + x}{(1+x)^2} = 0 \Rightarrow 1 + 2x + x^2 + x = 0 \Rightarrow x^2 + 3x + 1 = 0$

$\Rightarrow$ set numerator = 0 for x-int: Solve $x^2 + 3x + 1 = 0$ via quadratic formula

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2} \Rightarrow \frac{-3 + \sqrt{5}}{2} \approx -0.4$$

x-ints: $x = (-2.6, 0)$ and $(-0.4, 0)$

10.4 CN: $f'(x) = 0 \Rightarrow 1-x = 0 \Rightarrow x = 1$

$f'(x)$ undefined $\Rightarrow (1+x)^3 = 0 \Rightarrow x = -1$ ($f(x)$ and $f'(x)$ are undefined here)

$\Rightarrow$ Two critical numbers, but there is a critical point only at $x = 1$

10.5 CP: $(x_c, f(x_c)) = (1, f(1)) = (1, 1.25) = \text{CP}$

$$f(1) = 1 + \frac{1}{(1+1)^2} = 1 + \frac{1}{4} = \frac{5}{4} = 1.25$$

10.6 2nd derivative test: $f''(1) = \frac{2(1-2)}{(1+1)^4} = \frac{(-2)}{16} = -\frac{1}{8} \Rightarrow$ Rel Max

10.7 IPI: $f''(x) = \frac{2(x-2)}{(1+x)^4} = 0 \Rightarrow x = 2$ possible location of IP

Test: $f''(x) > 0$ for $x < 2$ and $f''(x) < 0$ for $x > 2$

$$f''(1.9) = \frac{2(1.9-2)}{(1+1.9)^4} = \frac{(-0.2)}{(2.9)^4} < 0$$

$$f''(2.1) = \frac{2(2.1-2)}{(1+2.1)^4} = \frac{(0.2)}{(3.1)^4} > 0$$

Concavity changes $\Rightarrow$ IP

IP is $(2, f(2)) = (2, 1/9)$

$$f(2) = 1 + \frac{2}{(3)^2} = 1 + \frac{2}{9} = \frac{11}{9}$$